Math 180 Written Homework

Assignment #9

Due **Tuesday, November 18th** at the beginning of your discussion class.

Directions. You are welcome to work on the following problems with other MATH 180 students, but your solutions must be hand-written, by your own hand, representing your understanding of the material. Word-by-word copying from another student or any other source is unacceptable. Any work without the proper justification will receive no credit. The list of problem solutions is to be submitted to your TA at the beginning of the discussion class listed above. No late homework will be accepted.

1. Compute the limit $\lim_{x\to 0^+} x^{\sin x}$.

2. Let $f(x) = \dfrac{1}{x}$ on $[1, 5]$; $n = 4$.

   (a) Illustrate the left and right Riemann sums for f on the given interval and for the given value of n. Determine which Riemann sum underestimates and which sum overestimates the area under the curve.

   (b) Calculate the left and right Riemann sums.

3. Each of the sums below is written in sigma notation. Expand it to a sum in which each term is written separately. (For example, $\displaystyle\sum_{n=1}^{4} 2n$ would become $2+4+6+8$.)

   (a) $\displaystyle\sum_{n=0}^{5} \frac{1}{(-2)^n}$

   (b) $\displaystyle\sum_{n=-3}^{3} \frac{n-1}{(n^2+1)}$

   (c) $\displaystyle\sum_{n=-1}^{6} (-n)^{n-2} \cos\left(\frac{n\pi}{2}\right)$

4. Identify a pattern in the terms of each sum written below and use the pattern to convert the sum to sigma notation. (For example, $2+4+6+8$ could become $\displaystyle\sum_{n=1}^{4} 2n$ or $\displaystyle\sum_{n=0}^{3} 2(n+1)$.)

   (a) $3+6+11+18+27+38+51+66+83+102+123+146+171+198+227+258+291+326+363+402$

   (b) $\dfrac{1}{9} - \dfrac{1}{3} + 1 - 3 + 9 - 27 + 81 - 243$

(c) $\dfrac{\ln(11)}{23} + \dfrac{\ln(13)}{28} + \dfrac{\ln(15)}{33} + \dfrac{\ln(17)}{38} + \dfrac{\ln(19)}{43} + \dfrac{\ln(21)}{48}$

Note: For a mass m moving along the x axis, the equations of motion give the acceleration $a(t)$, velocity $v(t)$ and the displacement $s(t)$ as a function of time t.

- acceleration is defined as $a(t) = \frac{dv}{dt}$
- velocity is defined as $v(t) = \frac{ds}{dt}$
- a, v and s can be positive (in the positive x direction) or negative (in the negative x direction).

5. A mass m is moving along the positive x direction with constant acceleration $a(t) = -9.8$.

   Find

   (a) $v(t)$ if $v(0) = 3$; and

   (b) $s(t)$ if $v(0) = 3$ and $s(0) = 4$

6. Mass m is moving along the x axis with acceleration

$$a = 2\cos(3t).$$

   Find

   (a) $v(t)$ if $v(0) = 4$; and

   (b) $s(t)$ if $v(0) = 4$ and $s(0) = 5$ the equations of motion:

7. Let R be the region bounded by the graph of $f(x) = \sin x$ and the x-axis between $x = 0$ and $x = \pi/2$.

   (a) Approximate the area of R using a left Riemann sum with $n = 3$ subintervals. Evaluate all trigonometric functions. Illustrate the sum with the appropriate rectangles.

   (b) Approximate the area of R using a right Riemann sum with $n = 3$ subintervals. Evaluate all trigonometric functions. Illustrate the sum with the appropriate rectangles.

   (c) How do the area approximations in parts (a) and (b) compare to the actual area under the curve?

8. Calculate the following indefinite integrals

   (a) $\displaystyle\int \csc^2 x \; dx$

   (b) $\displaystyle\int 2^t \; dt$

   (c) $\displaystyle\int \left(\sqrt{x} - 3\sqrt[5]{x}\right) \; dx$

   (d) $\displaystyle\int \left(e^{2t} + \frac{1}{2t}\right) \; dt$