

Math 180 Written Homework

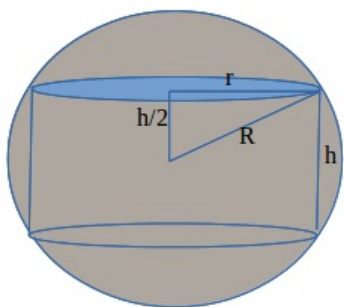
Assignment #8

Due **Tuesday, November 11th** at the beginning of your discussion class.

Directions. You are welcome to work on the following problems with other MATH 180 students, but your solutions must be hand-written, by your own hand, representing your understanding of the material. Word-by-word copying from another student or any other source is unacceptable. Any work without the proper justification will receive no credit. The list of problem solutions is to be submitted to your TA at the beginning of the discussion class listed above. No late homework will be accepted.

1. Let $f(x) = \ln(2/x)$.
 - (a) Compute the best linear approximation of f at $x = 2$.
 - (b) Use the best linear approximation from part (a) to estimate the value of $\ln(1.9)$.
 - (c) Determine whether your estimate of $\ln(1.9)$ in part (b) is an overestimate or an underestimate of the actual value of $\ln(1.9)$. Use calculus to justify your answer, not a calculator.
2. Sketch the graph of the function $f(x) = \frac{1}{3}x - (x - 1)^{1/3}$. Find and label all critical points, local maxima, local minima, inflections, and asymptotes.

Note: Your graph must include the y -intercept, but it is not necessary to find and label the x -intercepts. There are no horizontal asymptotes.
3. Find the volume of the largest right circular cylinder of radius r and height h that can be inscribed in a sphere of radius R .



4. The radius of a cylinder is decreasing at a rate of 4 ft/min, while the height is increasing at a rate of 2 ft/min. Find the rate of change in the volume when the radius is two feet and the height is six feet.

5. Ship A is steaming north at 10 miles per hour. Ship B, which is 5 miles west of ship A, is steaming east at 15 miles per hour.
- At what rate is the distance between them changing?
 - At what rate will it be changing one hour from now?
6. Consider the function $g(x) = -|x| + 1$ for $-1 \leq x \leq 1$.
- Draw g on the domain given.
 - A rectangle is to be formed with one side on the x -axis, and the vertices of the opposite side lying on g . Find the maximum area of such a rectangle.
7. How close does the semicircle $y = \sqrt{16 - x^2}$ come to the point $(1, \sqrt{3})$?
8. A marathon runner ran the Chicago marathon (which is 26.2 miles long) in 2 hours and 12 minutes. Explain (using calculus) how you know the runner had to be running at exactly 11 miles per hour at least twice during the race.
9. (a) Show that the best linear approximation of $f(x) = (1 + x)^k$ at $x = 0$ is $L(x) = 1 + kx$.
- Use your answer in part (a) to find the best linear approximation for the functions $f(x)$ at $x = 0$.
 - $f(x) = (1 - x)^6$
 - $f(x) = (4 + 3x)^{1/3}$
 - Using your answer in part (a), estimate $\sqrt[4]{1.009}$. Write your estimate in the form $\frac{a}{b}$ where a and b are integers.
10. Consider the function $h(x) = \frac{|x - 1|}{x + 2}$.
- Write h as a piecewise function.
 - State the domain of h .
 - Find and classify the critical points of h .
 - Find the intervals of increase and decrease for h .
 - Find any vertical asymptotes of h .
 - Find all horizontal asymptotes of h .
 - Using your information from the parts above, sketch a graph of h .