Math 180 Written Homework
Assignment #7
Due **Tuesday, October 21st** at the beginning of your discussion class.

Directions. You are welcome to work on the following problems with other MATH 180 students, but your solutions must be hand-written, by your own hand, representing your understanding of the material. Word-by-word copying from another student or any other source is unacceptable. Any work without the proper justification will receive no credit. The list of problem solutions is to be submitted to your TA at the beginning of the discussion class listed above. No late homework will be accepted.

1. Sketch the graph of a function that is continuous on $(-\infty, \infty)$ and satisfies all the following conditions:

   - $f'(x) > 0$ and $f''(x) > 0$ on $(-\infty, -2)$;
   - $f'(x) > 0$ and $f''(x) < 0$ on $4, \infty)$;
   - $f'(-2)$ and $f'(4)$ do not exist;
   - $f'(-1) = f'(1) = 0$ and $f''(0) = 0$.

2. Find the absolute maximum and minimum values of the function $f(x) = 3|x| - x^3$ on the interval $[-1, 2]$.

3. Let $f(x)$ be the function defined on the interval $[0, 2\pi]$, whose derivative is

$$f'(x) = (\sin x + 1)(2 \cos x + \sqrt{3}).$$

   (a) What are critical points of f?

   (b) Determine the intervals on which f is increasing and decreasing.

   (c) At what points, if any, does f have local maximum and local minimum values?

4. (a) Where is $f(x) = x - \ln x$ increasing?

   (b) Using part (a), prove that $\ln x \leq x$ for $x \geq 1$.

5. Let $f(x) = 3x^5 - 20x^3 + 14$.

   - Find the coordinates of the local extrema and horizontal points of inflection of f.
   - Find the intervals of concavity for f.

6. Find the absolute extrema of $g(x) = x^4 - 32x^2 - 7$ on $[-5, 6]$ (if it exists).

7. Find the absolute extrema of $h(x) = \dfrac{8 + x}{8 - x}$ on $[4, 6]$ (if it exists).