

## Math 180 Written Homework

### Assignment #3

Due **Tuesday, September 16th** at the beginning of your discussion class.

Directions. You are welcome to work on the following problems with other MATH 180 students, but your solutions must be hand-written, by your own hand, representing your understanding of the material. Word-by-word copying from another student or any other source is unacceptable. Any work without the proper justification will receive no credit. The list of problem solutions is to be submitted to your TA at the beginning of the discussion class listed above. No late homework will be accepted.

**In a limit problem where the limit does not exist because of an infinite limit, determine if the limit is  $+\infty$  or  $-\infty$  and justify your answer.**

1. Let  $f(x) = \frac{\sqrt[3]{x^6 + 729}}{4x^2 + \sqrt{2x^4 + 64}}$ .

(a) Find the domain of  $f$  and state the values where  $f$  is continuous. Justify your answers.

(b) Find  $\lim_{x \rightarrow -\infty} f(x)$ .

(c) Find  $\lim_{x \rightarrow +\infty} f(x)$ .

(d) Find all of the horizontal asymptotes of  $f(x)$ . If it has none, explain why not.

(e) Find all of the vertical asymptotes of  $f(x)$ . If it has none, explain why not.

2. Evaluate the following limits (justify your answers!):

$$\lim_{x \rightarrow -\infty} \frac{1}{\arctan x} \qquad \lim_{x \rightarrow +\infty} \frac{1}{\arctan x}.$$

3. Evaluate the following limits (justify your answers!):

$$\lim_{x \rightarrow -\infty} e^{-x/10} \qquad \lim_{x \rightarrow +\infty} e^{-x/10}.$$

4. Evaluate the following limits (justify your answers!):

$$\lim_{x \rightarrow -\infty} \frac{x}{\sqrt{x^2 + 1}} \qquad \lim_{x \rightarrow +\infty} \frac{x}{\sqrt{x^2 + 1}}.$$

5. Find all horizontal asymptotes of  $f(x) = \frac{\sin^2 x}{x}$ .

6. Consider the function  $g(x) = \cos(e^{\sqrt{x}})$ .

(a) State the domain of  $g$ .

(b) State the values (if any) in the domain of  $g$  where  $g$  is discontinuous. Justify your answer.

7. Find the values at which the function

$$f(x) = \begin{cases} 2x + 1 & \text{if } x \leq -1 \\ 3x & \text{if } -1 < x < 1 \\ -5x + 7 & \text{if } 1 \leq x < 2 \\ 2x - 7 & \text{if } x > 2 \end{cases}$$

is discontinuous. Sketch a graph of  $f$  to verify your discontinuities.

8. Find the derivatives of the following functions using the definition of the derivative (i.e. the limit of a difference quotient).

(a)  $x^3 + 2x$

(b)  $\frac{1}{\sqrt{t-3}}$

9. Sketch the graph of a function  $f(x)$  that is differentiable at every value of  $x$  except  $x = -3, 0, 5$ . Explain why you drew your graph the way you did.