Math 180 Written Homework Solutions

Assignment #2

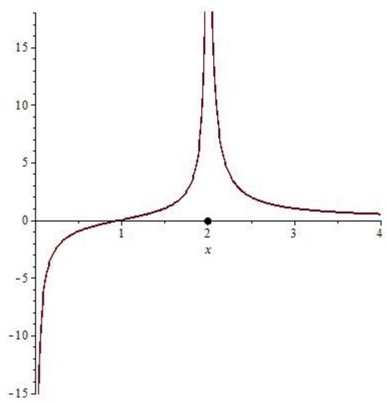
Due **Tuesday, September 9th** at the beginning of your discussion class.

Directions. You are welcome to work on the following problems with other MATH 180 students, but your solutions must be hand-written and by your own hand. The list of problem solutions is to be submitted to your TA at the beginning of the discussion class listed above. An answer without proper justification will receive little to no credit. No late homework will be accepted.

1. On a set of axes, draw one function $f(x)$ that has ALL of the following properties.

   - The domain of f is $x > 0$.
   - $\lim_{x\to 0^+} f(x) = -\infty$
   - $\lim_{x\to 2^-} f(x) = +\infty$
   - $\lim_{x\to 2^+} f(x) = +\infty$

   Solution: Here is one solution. There are more correct answers besides just this one.



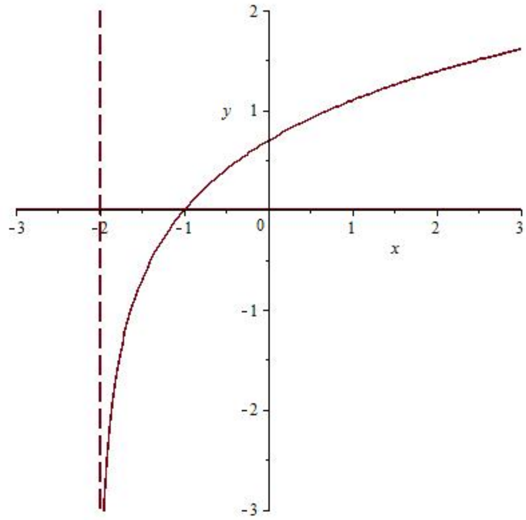
2. For each of the following limits, draw a sketch of the function near the indicated value and then find the limit based on your sketch.

   (a) $\displaystyle\lim_{x\to -2^+} \ln(x+2)$

   (b) $\displaystyle\lim_{x\to 0} \ln|x|$

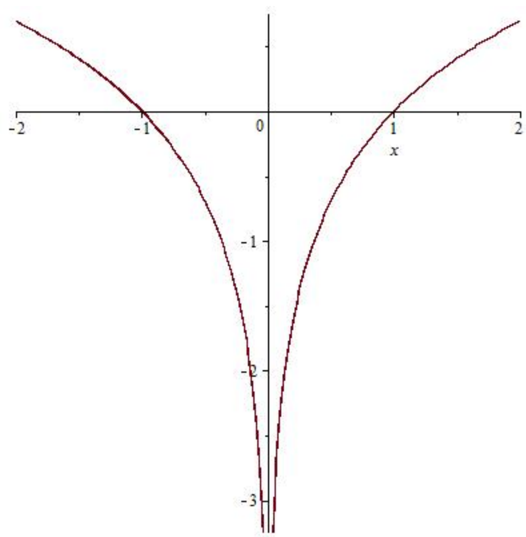
   (c) $\displaystyle\lim_{x\to 0} \frac{x}{e^x - 1}$

   Solution: (a)



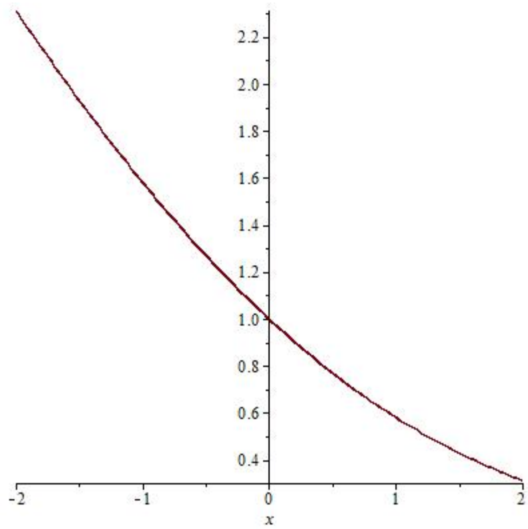
$$\lim_{x \to -2^+} \ln(x+2) = -\infty$$

(b)



$$\lim_{x \to 0} \ln|x| = -\infty$$

(c)



$$\lim_{x\to 0}\frac{x}{e^x-1}=1$$

3. Find all vertical asymptotes of the following functions.

   (a) $\ln(x+2)$

   (b) $\ln|x|$

   (c) $\dfrac{x}{e^x-1}$

   SOLUTION: (a) Since $\lim\limits_{x\to -2^+}\ln(x+2)=-\infty$, $x=-2$ is a vertical asymptote. Also since $\ln(x+2)$ is defined for every $x>-2$, there are no other vertical asymptotes.
   (b) Since $\lim\limits_{x\to 0}\ln|x|=-\infty$, $x=0$ is a vertical asymptote. Also since $\ln|x|$ is defined for every $x\neq 0$, there are no other vertical asymptotes.
   (c) Since $\dfrac{x}{e^x-1}$ is only undefined when $x=0$ and we showed $\lim\limits_{x\to 0}\dfrac{x}{e^x-1}=0$, there are no vertical asymptotes.

4. Evaluate the following limits.

   (a) $\lim\limits_{\theta\to 0^-}\theta^3\cos\left(\dfrac{2}{\theta}\right)$

   (b) $\lim\limits_{\theta\to 0^+}\theta^3\cos\left(\dfrac{2}{\theta}\right)$

SOLUTION: (a) We know $-1 \le \cos\left(\dfrac{2}{\theta}\right) \le 1$. If $\theta < 0$, then $\theta^3 < 0$ too so

$$-\theta^3 \ge \theta^3 \cos\left(\frac{2}{\theta}\right) \ge \theta^3$$

Then since

$$\lim_{\theta \to 0^-} \theta^3 = \lim_{\theta \to 0^-} \left(-\theta^3\right) = 0,$$

by the Squeeze Theorem

$$\lim_{\theta \to 0^-} \theta^3 \cos\left(\frac{2}{\theta}\right) = 0$$

(b) We know $-1 \le \cos\left(\dfrac{2}{\theta}\right) \le 1$. If $\theta > 0$, then $\theta^3 > 0$ too so

$$-\theta^3 \le \theta^3 \cos\left(\frac{2}{\theta}\right) \le \theta^3$$

Then since

$$\lim_{\theta \to 0^+} \left(-\theta^3\right) = \lim_{\theta \to 0^+} \theta^3 = 0,$$

by the Squeeze Theorem

$$\lim_{\theta \to 0^+} \theta^3 \cos\left(\frac{2}{\theta}\right) = 0$$

5. Evaluate $\displaystyle\lim_{x \to 1} \frac{x-1}{x^3-1}$ or state that it does not exist.

SOLUTION: $\displaystyle\lim_{x \to 1} \frac{x-1}{x^3-1} = \frac{0}{0}$ so we do algebra. We can factor $x^3 - 1$ as $(x-1)(x^2+x+1)$ so

$$\lim_{x \to 1} \frac{x-1}{x^3-1} = \lim_{x \to 1} \frac{x-1}{(x-1)(x^2+x+1)} = \lim_{x \to 1} \frac{1}{x^2+x+1} = \frac{1}{1+1+1} = \frac{1}{3}$$

6. Evaluate the following limit

$$\lim_{t \to 0} \frac{\dfrac{1}{2+\sin t} - \dfrac{1}{2}}{\sin t}$$

or state that it does not exist.

SOLUTION: $\lim_{t\to 0} \dfrac{\dfrac{1}{2+\sin t} - \dfrac{1}{2}}{\sin t} = \dfrac{0}{0}$ so we do algebra.

$$\begin{aligned}
\frac{\dfrac{1}{2+\sin t} - \dfrac{1}{2}}{\sin t} &= \frac{\dfrac{2}{2(2+\sin t)} - \dfrac{2+\sin t}{2(2+\sin t)}}{\sin t} = \frac{2-(2+\sin t)}{2(2+\sin t)} \cdot \frac{1}{\sin t} \\
&= \frac{-\sin t}{2(2+\sin t)} \cdot \frac{1}{\sin t} = -\frac{1}{2(2+\sin t)}
\end{aligned}$$

Now we take the limit

$$\lim_{t\to 0}\left(-\frac{1}{2(2+\sin t)}\right) = -\frac{1}{4}$$

Below are the problems that were graded and the scoring system that was used for each problem.

1. [10 points] On a set of axes, draw one function $f(x)$ that has ALL of the following properties.

- The domain of f is $x > 0$.
- $\lim_{x\to 0^+} f(x) = -\infty$
- $\lim_{x\to 2^-} f(x) = +\infty$
- $\lim_{x\to 2^+} f(x) = +\infty$

2 points – If a student properly drew a graph with domain $x > 0$

2 points – If a student properly drew a graph with the right-hand limit at 0 equaling $-\infty$

2 points – If a student properly drew a graph with the left-hand limit at 2 equaling $+\infty$

2 points – If a student properly drew a graph with the right-hand limit at 2 equaling $+\infty$

2 points – Only if a student got each of the four bullet points correct

Note: Each of the bullet points is worth either 0 or 2 points; it's only 2 points if it is completely correct, otherwise 0

5. [15 points] Evaluate $\displaystyle\lim_{x\to 1} \frac{x-1}{x^3-1}$ or state that it does not exist.

0 points – If a student uses a graph or a table of values to compute the limit

2 points – If a student got a $\frac{0}{0}$ limit by simply plugging in $x = 1$, but did nothing else

10 points – If a student properly factors $x^3 - 1$ and cancels the $x - 1$ from both, resulting in $\dfrac{1}{x^2+x+1}$

15 points – Only if a student properly factors and cancels and plugs in $x = 1$ to get the limit $\dfrac{1}{3}$

Note: If a student makes an algebra mistake before evaluating the limit, but completes the rest of the problem correctly, that student can earn at most 6 points total on the problem.

6. [15 points] Evaluate the following limit

$$\lim_{t\to 0} \frac{\dfrac{1}{2+\sin t} - \dfrac{1}{2}}{\sin t}$$

or state that it does not exist.

0 points – If a student uses a graph or a table of values to compute the limit

2 points – If a student got a $\frac{0}{0}$ limit by simply plugging in $t = 0$, but did nothing else

10 points – If a student simplifies the rational expression correctly to $-\dfrac{1}{2(2+\sin t)}$

15 points – Only if a student properly simplified the expression and plugged in $t = 0$ to get a limit of $-\frac{1}{4}$

Note: If a student makes an algebra mistake before evaluating the limit, but completes the rest of the problem correctly, that student can earn at most 6 points total on the problem.