Math 180 Written Homework Solutions

Assignment #1

Due **Thursday, September 4th** at the beginning of your discussion class.

Directions. You are welcome to work on the following problems with other MATH 180 students, but your solutions must be hand-written and by your own hand. The list of problem solutions is to be submitted to your TA at the beginning of the discussion class listed above. No late homework will be accepted.

1. Write down the names (first and last) of at least five other MATH 180 students. They do not have to be from your lecture or discussion. We suggest getting their contact information too so you can work on future homework assignments with them (and others) as the semester progresses.

2. Evaluate $\lim\limits_{x\to 7} \dfrac{\frac{3}{x}-\frac{3}{7}}{x-7}$

   Solution: $\lim\limits_{x\to 7} \dfrac{\frac{3}{x}-\frac{3}{7}}{x-7} = \dfrac{0}{0}$ so we do algebra.

$$\begin{aligned}\lim_{x\to 7} \frac{\frac{3}{x}-\frac{3}{7}}{x-7} &= \lim_{x\to 7} \frac{\frac{21}{7x}-\frac{3x}{7x}}{x-7} = \lim_{x\to 7} \frac{\frac{21-3x}{7x}}{x-7} \\ &= \lim_{x\to 7} \frac{-3(x-7)}{7x}\cdot\frac{1}{x-7} = \lim_{x\to 7}\left(-\frac{3}{7x}\right) \\ &= -\frac{3}{49}\end{aligned}$$

3. Evaluate $\lim\limits_{x\to -3} \dfrac{x+3}{\sqrt{84+x}-9}$

   Solution: $\lim\limits_{x\to -3} \dfrac{x+3}{\sqrt{84+x}-9} = \dfrac{0}{0}$ so we do algebra.

$$\begin{aligned}\frac{x+3}{\sqrt{84+x}-9} &= \frac{x+3}{\sqrt{84+x}-9}\cdot\frac{\sqrt{84+x}+9}{\sqrt{84+x}+9} = \frac{(x+3)\left(\sqrt{84+x+9}\right)}{84+x-81} \\ &= \frac{(x+3)\left(\sqrt{84+x+9}\right)}{x+3} = \sqrt{84+x}+9\end{aligned}$$

   Now we take the limit

$$\lim_{x\to -3}\left(\sqrt{84+x}+9\right) = \sqrt{84+(-3)}+9 = 18$$

4. Consider the function

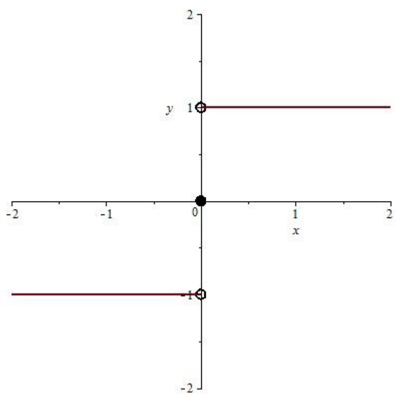
$$f(x) = \begin{cases} \dfrac{|x|}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0. \end{cases}$$

(a) Sketch a graph of f on the interval $[-2, 2]$.

(b) Determine $\lim_{x \to 0} f(x)$. Justify your answer.

(c) Determine $\lim_{x \to 1} f(x)$. Justify your answer.

SOLUTION: (a)



(b) $\lim_{x \to 0} f(x)$ does not exist since from the graph the left-hand limit is -1 and the right-hand limit is 1.

(c) $\lim_{x \to 1} f(x) = 1$ since from the graph both the left- and right-hand limits are 1.

5. The *floor* function floor(x) calculates the largest integer less than or equal to x, and the *ceiling* function ceiling(x) calculates the smallest integer greater than or equal to x. Here are some examples:

$$\begin{gathered} \text{floor}(4.5) = 4 \\ \text{floor}(-2.8) = -3 \\ \text{floor}(8) = 8 \\ \text{ceiling}(6.1) = 7 \\ \text{ceiling}(-1.99) = -1 \end{gathered}$$

Now define a function by

$$g(x) = \frac{\text{ceiling}(x)}{\text{floor}(x)}.$$

(a) What is the domain of g?

(b) Graph g on the interval $[-5, 5]$. [You might want to use graph paper for this graph.]

(c) Calculate the following limits or state that they don't exist.

i. $\lim\limits_{x\to 2} g(x)$

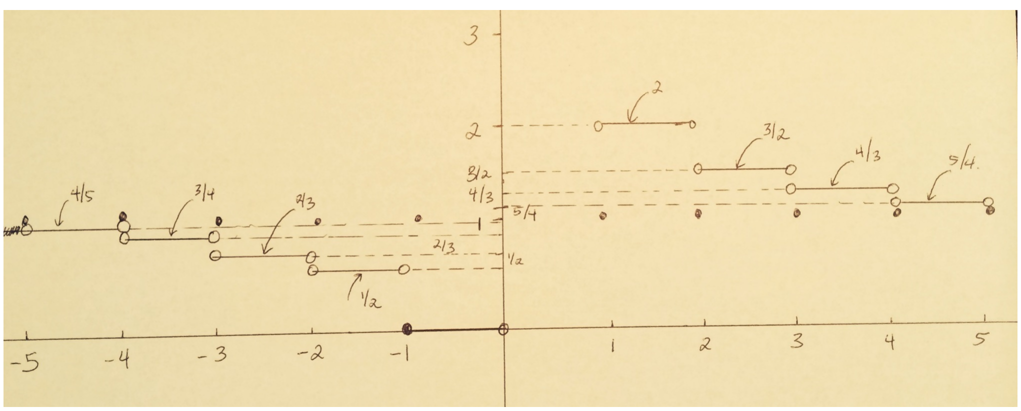
ii. $\lim\limits_{x\to \frac{5}{2}} g(x)$

iii. $\lim\limits_{x\to 0^-} g(x)$

(d) Find a value for c such that $\lim\limits_{x\to c} g(x) = \dfrac{3}{2}$.

SOLUTION: (a) The domain of g is all reals except $0 \le x < 1$ since the values $0 \le x < 1$ make floor$(x) = 0$.

(b)



(c)

(i) From the graph, $\lim\limits_{x\to 2^-} g(x) = \dfrac{2}{1}$ and $\lim\limits_{x\to 2+} g(x) = \dfrac{3}{2}$, so $\lim\limits_{x\to 2} g(x)$ does not exist.

(ii) From the graph, $\lim\limits_{x\to \frac{5}{2}^-} g(x) = \lim\limits_{x\to \frac{5}{2}^+} g(x) = \dfrac{3}{2}$, so $\lim\limits_{x\to \frac{5}{2}} g(x) = \dfrac{3}{2}$.

(iii) From the graph, all values to the left of 0 have corresponding y-values equal to 0, so $\lim\limits_{x\to 0^-} g(x) = 0$

(d) From part (b), we notice that $\lim\limits_{x\to \frac{5}{2}} g(x) = \dfrac{3}{2}$, so one value for c could be $c = \dfrac{5}{2}$. [In fact every value $2 < c < 3$ would work.]

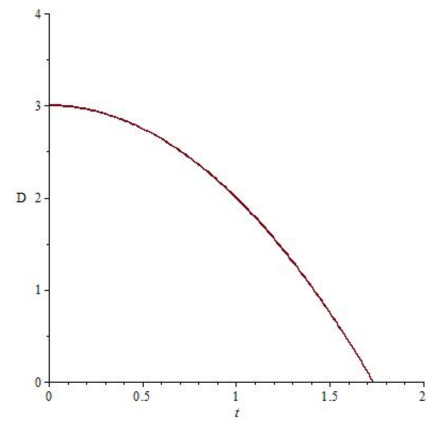
6. Consider the graphs below, where t represents time and D is distance. Assume the units on each axis are the same for all graphs.

   (a) Which graph represents an object that is slowing down? Explain your answer.

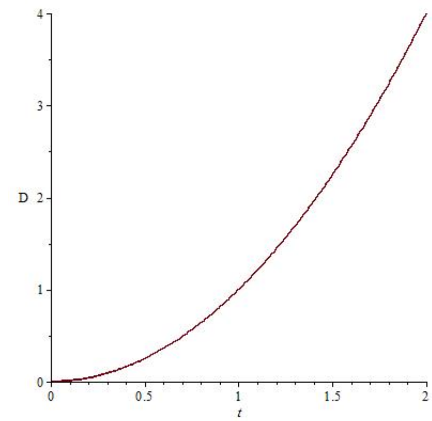
   (b) Which graph represents an object that is traveling at a constant speed? Explain your answer.

   (c) Which graph represents an object whose ***velocity*** is decreasing the fastest? Explain your answer.

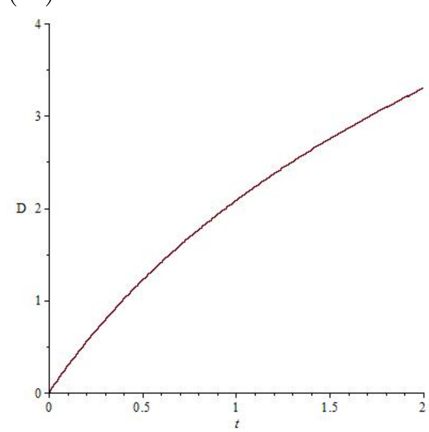
(i)



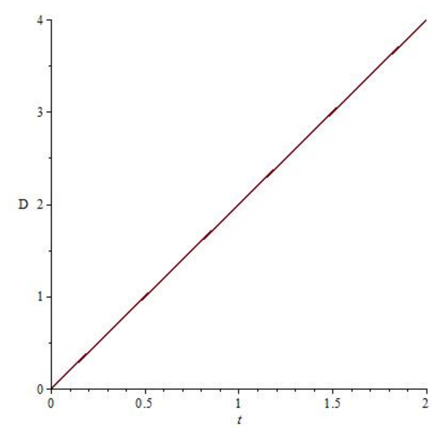
(ii)



(iii)



(iv)



Solution: (a) The graph in (iii) represents an object that is slowing down because as t increases, the absolute value of the slopes of the tangent lines are decreasing.

(b) The graph in (iv) represents an object that is traveling at a constant speed because as t increases, the slopes of the tangent lines are all the same.

(c) The graph in (i) represents an object whose velocity is decreasing the fastest because as t increases, the slopes of the tangent lines are getting smaller the quickest.

Below are the problems that were graded and the scoring system that was used for each problem.

3. [15 points] $\lim_{x\to -3} \dfrac{x+3}{\sqrt{84+x}-9}$

0 points – If a student uses L'Hopital's rule

2 points – If a student recognizes that simply plugging in -3 results in a $\frac{0}{0}$ limit and does nothing else

10 points – If a student properly multiplies numerator and denominator by the conjugate of the denominator and simplifies correctly to $\sqrt{84+x}+9$

15 points – Only if the student properly rationalizes to $\sqrt{84+x}+9$ and evaluates the limit to be 18

Note: If a student makes an algebra mistake before evaluating the limit, but completes the rest of the problem correctly, that student can earn at most 6 points total on the problem.

If a student chooses to use a table to calculate the limit, please write some version of the following comment "In the future (on homework and exams), tables can only be used to estimate the value of a limit and cannot be used to calculate the exact value." and use the following scale:

15 points – If a student chooses values sufficiently close to -3 on both sides and has the correct corresponding y-values which make it "obvious" to determine the limit is 18

5 points – If a student only completes a table of values on one side of -3 (again the table must have values properly chosen near -3 and the associated y-values must be correct)

0 points – Otherwise

5. [20 points] The *floor* function floor(x) calculates the largest integer less than or equal to x, and the *ceiling* function ceiling(x) calculates the smallest integer greater than or equal to x. Define a function by

$$g(x) = \frac{\text{ceiling}(x)}{\text{floor}(x)}.$$

(a) What is the domain of g?

(b) Graph g on the interval $[-5, 5]$. [You might want to use graph paper for this graph.]

(c) Calculate the following limits or state that they don't exist.

i. $\lim_{x\to 2} g(x)$

ii. $\lim_{x \to \frac{5}{2}} g(x)$

iii. $\lim_{x \to 0^-} g(x)$

(d) Find a value for c such that $\lim_{x \to c} g(x) = \dfrac{3}{2}$.

Part 1. 2 points – If a student has the correct domain

0 points – If a student does not have the correct domain

Part 2. 5 points – If a student has the correct graph (this includes endpoints of each step of the graph)

2 points – If a student has certain parts of the graph correct, but the complete graph is not correct

0 points – If a student's graph does not resemble the graph at all

Part 3. Each of parts (a), (b), and (c) are worth 3 points.

3 points – If a student has the correct answer AND the answer is supported by his graph

0 points – Otherwise

Part 4. 4 points – If a student has a correct value for c AND the answer is supported by his graph

0 points – Otherwise