Math 180 Written Homework

Assignment #10

Due **Tuesday, December 2nd** at the beginning of your discussion class.

Directions. You are welcome to work on the following problems with other MATH 180 students, but your solutions must be hand-written, by your own hand, representing your understanding of the material. Word-by-word copying from another student or any other source is unacceptable. Any work without the proper justification will receive no credit. The list of problem solutions is to be submitted to your TA at the beginning of the discussion class listed above. No late homework will be accepted.

1. For each definite integral draw a picture of the net area the integral is representing. Then calculate the integral (using any method you choose).

   (a) $\displaystyle\int_1^3 (2x+1)\ dx$

   (b) $\displaystyle\int_{-1}^1 (-2|x|+1)\ dx$

   (c) $\displaystyle\int_0^9 \sqrt{9-t^2}\ dt$

2. Suppose that $f(x) \geq 0$ on $[0,3]$, $f(x) \leq 0$ on $[3,7]$, $\displaystyle\int_0^3 f(x)\ dx = 4$ and $\displaystyle\int_3^7 f(x)\ dx = -10$.

   (a) Sketch a possible graph of f on the interval $[0,7]$.

   (b) Find $\displaystyle\int_0^7 f(x)\ dx$.

   (c) Using your sketch from (a), sketch $|f(x)|$ on $[0,7]$.

   (d) Find $\displaystyle\int_0^7 |f(x)|\ dx$

   (e) Find $\displaystyle\int_0^7 (3f(x) - 2|f(x)|)\ dx$

3. Let $f(x) = \sqrt{24 - 2x - x^2}$.

   (a) What is the domain of $f(x)$?

   (b) Without using a calculator, graph $f(x)$. [Hint: Complete the square.]

   (c) Using geometry, calculate $\displaystyle\int_{-6}^4 f(x)\ dx$. [Note: You may not use the Fundamental Theorem of Calculus; you must use geometry.]

4. Let $F(x) = \int_{x^3}^{5} \left(\cos^2 t - te^t\right)\ dt$. Find $F'(x)$

5. Calculate the following integrals.

   (a) $\int \sin x \cos x\ dx$

   (b) $\int \sin^2 t\ dt$ [Hint: $1 - 2\sin^2 t = \cos(2t)$]

   (c) $\int_0^{\sqrt{\pi}} xe^{x^2}\ dx$

   (d) $\int_{e^2}^{e^4} \frac{3}{x \ln x}\ dx$

   (e) $\int x\sqrt{x+2}\ dx$

6. Find two values for c such that $\int_0^c (x^2 - 2x)\ dx = 0$

7. Let $y = (x-1)^2$.

   (a) Find the average value of y on $[0, 3]$.

   (b) Find all points in $[0, 3]$ where f attains the average value you found in (a).