

## Correction to example from Lecture 25 (Oct 22)

**Problem:** Suppose  $f(x)$  is a function satisfying

- $f(0) = 1$
- $f$  is increasing on  $(-\infty, 1)$  and  $(3, \infty)$
- $f$  has a local maximum at  $x = 1$
- $f$  has a local minimum at  $x = 3$
- $f$  is concave up on  $(-\infty, -1)$  and  $(2, \infty)$

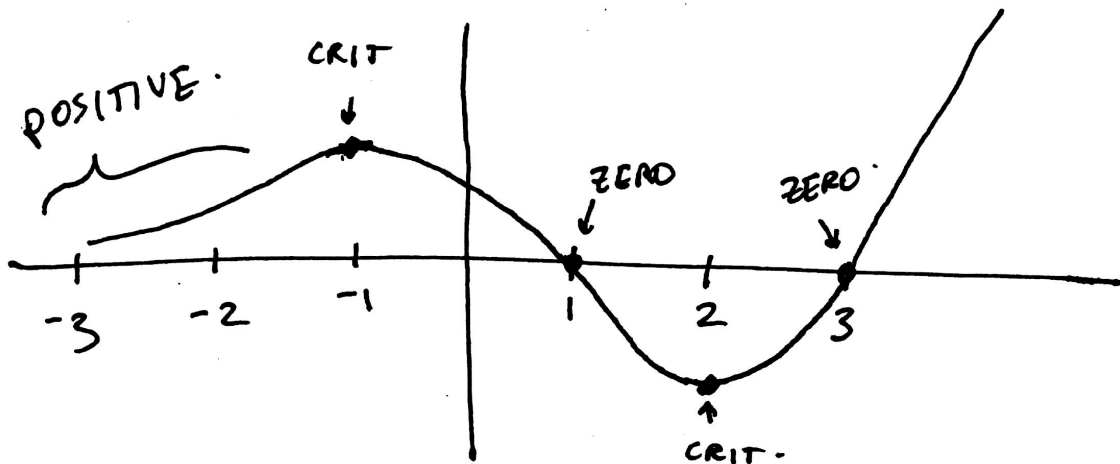
Let  $g(x) = f'(x)$ . Sketch what the graph of  $g(x)$  might look like.

**Solution:**

We translate the statements into properties of  $g(x) = f'(x)$ :

- $f(0) = 1$  says nothing about  $g(x)$
- $g$  is positive on  $(-\infty, 1)$  and  $(3, \infty)$
- $g$  changes sign from positive to negative at  $x = 1$
- $g$  changes sign from negative to positive at  $x = 3$
- $g$  is increasing on  $(-\infty, -1)$  and  $(2, \infty)$

Here is a graph that has these properties (which would be an acceptable solution to the problem):



In lecture on October 22, I drew a similar figure but incorrectly had the graph of  $g$  crossing the  $x$ -axis at some negative value of  $x$ . This is not consistent with the given information because  $g$  is supposed to be positive for all  $x < 1$ .

(Note: The corresponding graph of  $f(x)$  would look something like the picture below. The problem did not ask for this, but it might be helpful in understanding the image above.)

