Math 535 - Complex Analysis Midterm Exam

Advice: If you find yourself writing page after page of complicated calculations, you might want to stop and reconsider your approach. All of these problems have relatively "short" solutions.

(1) Let γ be the circle |z| = 535 oriented counter-clockwise. Calculate:

$$\int_{\gamma} \frac{\cos(z/2)}{4z^2 - \pi^2} \, dz$$

- (2) Suppose f(z) has a zero of order 2 at z = 0. Show that there is an analytic function g(z) defined in a neighborhood of the origin such that $f(z) = g(z)^2$ on their common domain.
- (3) Show that a nonconstant entire function has a negative real value at some point.