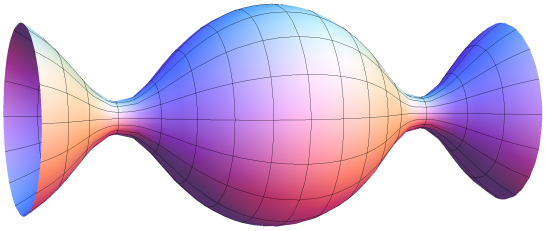
Math 442 - Differential Geometry of Curves and Surfaces

Fall 2010



1. General Information

Web Page	http://www.math.uic.edu/~ddumas/math442/
Textbook	Manfredo do Carmo, *Differential Geometry of Curves and Surfaces* Prentice-Hall, 1976. ISBN-13: 978-0-13-212589-5
Meeting Time	MWF 11:00-11:50am
Location	Stevenson Hall 215
CRN	32336 (undergrad) 32337 (grad)
Instructor	David Dumas (ddumas@math.uic.edu)
Office Hours	Mondays and Wednesdays, 2-3pm in SEO 503 and by appointment

2. Course Overview

The main objects of study in multivariable calculus are functions and maps on Euclidean space, $\mathbb{R}^n$. Differential geometry consists of an extension of multivariable calculus to more general spaces, *differentiable manifolds*, and the application of these tools to geometric problems.

In this course we will introduce differential geometry by studying the most classical examples of manifolds—regular curves and surfaces in three-dimensional Euclidean space. The first section of the course will center around the development of a local coordinate system adapted to a space curve, the *Frenet frame*, and understanding how it relates to geometric notions such as curvature, twisting, and tangent lines. We will also use differential techniques to establish some interesting global properties of curves in the plane, including the classical result that among smooth closed curves of a given arc length, a circle encloses the largest area.

We will then move on to regular surfaces in space. The central ideas in this section are the *induced metric*, whereby a surface inherits a 2-dimensional metric geometry from the ambient 3-dimensional space, and the *curvature*, which measures the way in which a small section of the surface deviates from a plane. These ideas are encoded in the *first and second fundamental forms* of a surface. Our study of surfaces will culminate with the Gauss-Bonnet theorem, which says that that the integral curvature of a closed surface in space determines its topological type (e.g. sphere, torus, ...). This is an example of an important principle in differential geometry: Local geometric information about a space can be used to understand its global properties.

3. Topic Outline

(1) Curves
   (a) Parameterized curves in $\mathbb{R}^3$, regularity, arc length
   (b) Curvature and torsion of space curves
   (c) The Frenet frame and Frenet-Serret equations
   (d) Plane curves: Cauchy-Crofton formula, isoperimetric inequality

(2) Surfaces
   (a) Regularity, examples
   (b) Calculus on surfaces: Change of variables, differentiable functions
   (c) Tangent planes and normal lines
   (d) Introducing metric geometry: the first fundamental form
   (e) Normal curvature, the Gauss map, and the second fundamental form
   (f) Gaussian curvature, mean curvature, and minimal surfaces
   (g) Isometries between surfaces
   (h) Gaussian curvature and the *Theorema Egregium*
   (i) Geodesics and geodesic curvature
   (j) Gauss-Bonnet theorem (local version)
   (k) Geodesic triangulations, topological classification of compact surfaces
   (l) Gauss-Bonnet theorem (global version)

4. Grading

Your final grade for the course will be based on your homework assignments, an in-class midterm exam, and a cumulative final exam. These components will be weighted as follows:

Homework		30%
Midterm	Wed, Oct 13	30%
Final Exam	Thurs, Dec 9	40%

5. Homework policies

There are two types of homework: Weekly problems and challenge problems.

Weekly homework will be assigned on the course web page, and will consist mostly of problems from the textbook. The weekly assignments are due by 4pm on the day of the first lecture of each week (usually a Monday), and can be turned in during lecture or directly to the grader's mailbox (Jonah Gaster) on the third floor of SEO.

A list of challenge problems is posted on the course web page and will be updated regularly. During the semester you are required to complete at least **four** of these problems and submit your written solutions, and at least two challenge problems must be submitted before the midterm exam. You are encouraged to complete and turn in more than the required number of challenge problems. Doing so will have a modest positive effect on your homework and course grades (in a manner to be determined at the end of the semester).

You are allowed and encouraged to study the course material and work on the homework with other students. However, you must:

(1) Write and submit your own solutions
(2) Acknowledge your collaborators by name on your assignment
(e.g. write "in collaboration with Jane Doe" at the top of the page).

Your homework grade will be determined by dropping your lowest weekly assignment score and then averaging the remaining weekly scores and your four required challenge problems. In particular, *each challenge problem is worth as much as a weekly homework assignment.*

6. Submitting challenge problems

Challenge problems must be submitted **in lecture** and are handled separately from weekly homework.

Write your solution to a challenge problem on a separate sheet of paper; if your solution to a single problem spans several pages, staple them together. When submitting a given problem for the first time, write your name, the problem number, and the words "first submission" on the top of the first page.

Your solution will be held to a high standard of completeness, clarity, and correctness; do not be alarmed if your solution is returned with comments and resubmission is requested.

To resubmit a problem, write your corrected solution on a new sheet of paper and staple it on *top* of all previously submitted versions Write your name, the problem number, and the word "resubmission" on the top of the first page.

Resubmission of challenge problems gives you an opportunity to correct mistakes and increase your score. At the end of the semester, only your highest score on each problem will be considered. There is no limit to the number of times you can submit a given problem, except for the practical limitation imposed by the time it takes to grade and return the challenge problems. You are encouraged to submit challenge problems early in the semester to maximize the time available to revise and resubmit them.

7. Attendance

Attending the lectures is mandatory. If you must miss a lecture, you should make arrangements to get notes and any class materials from someone else in the class. You are responsible for the contents of all lectures, including any that you cannot attend.

8. Academic honesty

All UIC students are expected to maintain the standards of academic honesty described in the *Guidelines for Academic Integrity* available from the Office of the Vice Chancellor for Student Affairs web page:

http://www.vcsa.uic.edu/MainSite/departments/dean_of_students/Our+Services/Student+Judicial+Affairs.htm#19

In particular, this policy prohibits plagiarism and giving or receiving aid on an examination.