

## Midterm Exam

Write your name and “Math 442 Fall 2010” on an exam book.

Complete **three** of the problems below.

In your solutions you can use propositions from the lectures, reading, or homework, but you must clearly explain what fact you are using and how it fits into your solution.

- (1) Prove that if two regular surfaces intersect at only one point, then they have the same tangent plane at that point. (That is, if  $S_1 \cap S_2 = \{p\}$  then  $T_p S_1 = T_p S_2$ .)

- (2) Determine the set of all positive real numbers  $A$  such that the equation

$$(x + y + z)^3 = A(x^3 + y^3 + z^3)$$

defines a regular surface in  $\mathbb{R}^3 - \{(0, 0, 0)\}$ .

- (3) (a) Define the torsion function  $\tau$  of a space curve.  
(b) Let  $\alpha : I \rightarrow \mathbb{R}^3$  denote a regular parameterized space curve without inflection points. Show that  $\alpha(I)$  lies in a plane if and only if the torsion of  $\alpha$  is identically zero.

- (4) (a) Define the curvature function  $\kappa$  of a plane curve.  
(b) Determine the curvature function of the *cycloid*

$$\alpha(t) = (at - b \sin(t), a - b \cos(t))$$

where  $a, b \in \mathbb{R}$  are constants and  $a \neq 0$ .