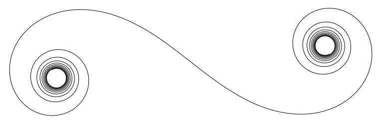
Math 442 - Differential Geometry of Curves and Surfaces

Spring 2009



1. General Information

Web Page	`http://www.math.uic.edu/~ddumas/math442/`
Textbook	Manfredo do Carmo, *Differential Geometry of Curves and Surfaces* Prentice-Hall, 1976. ISBN-13: 978-0-13-212589-5
Meeting Time	MWF 11:00-11:50am
Location	Stevenson Hall 215
CRN	32336 (undergrad) 32337 (grad)
Instructor	Emily Dumas (`ddumas@uic.edu`)
Office	SEO 503
Office Hours	Mon 2:00-3:00pm, Wed 11:00am-12:00pm and by appointment

2. Course Overview

The main objects of study in multivariable calculus are functions and maps on Euclidean space, $\mathbb{R}^n$. Differential geometry consists of an extension of multivariable calculus to more general spaces, *differentiable manifolds*, and the application of these tools to geometric problems.

In this course we will introduce differential geometry by studying the most classical examples of manifolds—regular curves and surfaces in three-dimensional Euclidean space. The first section of the course will center around the development of a local coordinate system adapted to a space curve, the *Frenet frame*, and understanding how it relates to geometric notions such as curvature, twisting, and tangent lines. We will also use differential techniques to establish some interesting global properties of curves in the plane, including the classical result that among smooth closed curves of a given arc length, a circle encloses the largest area.

We will then move on to regular surfaces in space. The central ideas in this section are the *induced metric*, whereby a surface inherits a 2-dimensional metric geometry from the ambient 3-dimensional space, and the *curvature*, which measures the way in which a small section of the surface deviates from a plane. These ideas are encoded in the *first and second fundamental forms* of a surface. Our study of surfaces will culminate with the Gauss-Bonnet theorem, which says that that the integral curvature of a closed surface in space determines its topological type (e.g. sphere, torus, ...). This is an example of an important principle in differential geometry: Local geometric information about a space can be used to understand its global properties.

3. Topics

(1) Curves
   (a) Parameterized curves in $\mathbb{R}^3$, regularity, arc length
   (b) Local invariants of curves and the Frenet frame
   (c) Plane curves: Cauchy-Crofton formula, isoperimetric inequality

(2) Surfaces
   (a) Regularity, examples
   (b) Calculus on surfaces: Change of variables, differentiable functions
   (c) Tangent planes and normal lines
   (d) Introducing metric geometry: the first fundamental form
   (e) Normal curvature, the Gauss map, and the second fundamental form
   (f) Isometries between surfaces
   (g) Gaussian curvature and the *Theorema Egregium*
   (h) Geodesics and geodesic curvature
   (i) The Gauss-Bonnet theorem

4. Grading

Your final grade for the course will be based on your homework assignments, an in-class midterm exam, and a cumulative final exam. These components will be weighted as follows:

Homework		30%
Midterm	Wed, Mar 4	30%
Final Exam	Fri, May 8	40%

5. Homework Policies

There are two types of homework: Weekly problems and challenge problems.

Weekly homework will be assigned on the course webpage, and will consist mostly of problems from the textbook. The weekly assignments will be collected in the first lecture of each week of the semester (which is usually a Monday).

A list of challenge problems will also be posted on the course webpage and updated regularly. During the semester, you are required to complete at least **two** of these problems and submit your written solutions in lecture. (Graduate students taking the course for credit must complete four challenge problems.) You can submit solutions to challenge problems at any time. Your solutions to these problems will be held to a high standard of completeness, clarity, and correctness; do not be alarmed if your solution is returned with comments and resubmission is requested.

You are allowed (and encouraged) to study the course material and work on the homework with other students. However, you must:

(1) Write and submit your own solutions
(2) Acknowledge your collaborators by name on your assignment
   (e.g. write "in collaboration with Jane Doe" at the top of the page).

Your homework grade will be determined by dropping your lowest weekly assignment score and then averaging the remaining weekly scores and your two required challenge problems. In particular, *each challenge problem is worth as much as a weekly homework assignment.*

You are encouraged to complete and turn in more than the required number of challenge problems. Doing so will have a modest positive effect on your homework grade in a manner to be determined before the end of the semester.

6. Attendance

Attending the lectures is mandatory. If you absolutely must miss a lecture, you should make arrangements to get notes and any class materials from someone else in the class. You are responsible for the contents of all lectures, including any that you cannot attend.

7. Academic Honesty

All UIC students are expected to maintain the standards of academic honesty described in the *Guidelines for Academic Integrity* available from the Office of the Vice Chancellor for Student Affairs web page:

http://www.vcsa.uic.edu/MainSite/departments/dean_of_students/Our+Services/Student+Judicial+Affairs.htm#19

In particular, this policy prohibits plagiarism and giving or receiving aid on an examination.

Any violation of these standards will be referred to the Office of Student Judicial Affairs, and severe penalties (such as temporary or permanent expulsion from the University) may result.