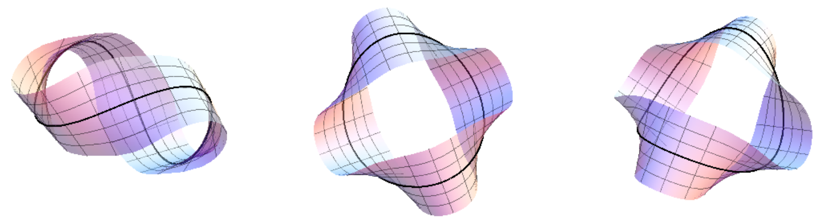
Math 442 / Emily Dumas / Spring 2009

Final Exam Problem List A

The math 442 final exam consists of two problem lists ("A" and "B") with four problems each. This is list "A". You will receive the second list on the day of the exam. For full credit you must solve two problems from each list during the two-hour exam period.

(A1) Give an example of a regular surface $S \subset \mathbb{R}^3$ and a simple closed geodesic $\gamma \subset A$ that is *not* contained in a plane. (Parameterize both S and γ, and prove that they have the requested properties.)

Hint: Such a surface and geodesic are shown in the pictures below.



(A2) Define the evolute and involute of a plane curve. Then state and prove a theorem that is a precise form of the statement "evolute and involute are inverse operations".

(A3) Suppose γ is a smooth closed curve that lies on the unit sphere in $\mathbb{R}^3$.

(a) Show that the curvature of γ satisfies $\kappa(s) \geq 1$.

(b) Show that at a point of maximum curvature, the torsion of γ vanishes.

(A4) Gauss's *Theorema Egregium* states that locally isometric surfaces have the same gaussian curvature at corresponding points. (That is, if $\phi : S \to \bar{S}$ is a local isometry, then $K(p) = \bar{K}(\phi(p))$.) Is the same true for mean curvature? Prove it or give a counterexample.