05200101

Math 520 Exam 1

Spring 2008

Sections 1–3 (Xiao/Dumas/Liaw)

Read these instructions carefully.

- Write your name, section number, and "Math 520 Exam 1" on the front of a blue exam book.

10am / Xiao	=	Section 1
Noon / Dumas	=	Section 2
1pm / Liaw	=	Section 3

- Read each problem carefully before you attempt to solve it.
- Write your solutions to the problems in the examination book.
- Clearly indicate where your solution to each problem begins and ends.
- Make sure your solutions are clear, concise, and legible.
- Each of the **5** problems will contribute approximately the same amount to your exam grade.
- **Manage your time carefully. If you get stuck on one problem, move on to another.**

Do not turn the page until you are told to do so!

1. Compute each product of matrices, or indicate that the matrices are not compatible for multiplication:

   (a) $\begin{pmatrix} 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}$

   (b) $\begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \end{pmatrix}$

   (c) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 3 & 1 \\ 5 & 2 & 5 \end{pmatrix}$

   (d) $\begin{pmatrix} 1 & 3 & 1 \\ 5 & 2 & 5 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

   (e) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

2. This question concerns the matrix

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & -1 \\ 2 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix}.$$

   (a) Compute R, the row reduced echelon form of A.

   (b) What is the rank of A?

   (c) Which columns are pivot columns, and which are free columns (if any)?

   (d) Find the special solutions to $A\boldsymbol{x} = \mathbf{0}$.

   (e) Compute the general solution to $A\boldsymbol{x} = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \end{pmatrix}$.

3. This question concerns the matrix

$$B = \begin{pmatrix} 1 & 2 & -5 \\ 0 & -1 & 2 \\ 4 & 10 & -23 \end{pmatrix}.$$

   (a) Find an invertible square matrix E such that $EB = U$ is upper triangular.

   (b) Compute the LU decomposition of B.

   (c) Compute the inverse matrix B^{-1}.

$* * *$ **The exam continues on the next page.** $* * *$

4. Let $\mathbf{M}_{2\times 3}$ denote the vector space of all 2×3 matrices.

   (a) Let $V \subset \mathbf{M}_{2\times 3}$ denote the set of matrices whose third column is $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$.
   Is V a subspace of $\mathbf{M}_{2\times 3}$? Either show that it is, or explain why it is not.

   (b) Describe the span of the following vectors in $\mathbf{M}_{2\times 3}$:

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

   (That is, describe it in a more useful way than "the span of the following vectors...".)

   (c) Let $W \subset \mathbf{M}_{2\times 3}$ denote the set of matrices A such that the null space of A contains $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

   Show that W is a subspace of $\mathbf{M}_{2\times 3}$.

5. (a) What does it mean for a set of vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n$ to be *linearly dependent*? (Write a definition.)

   (b) Show (directly) that these four vectors are linearly dependent:

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

   (c) Explain why *any* four vectors in $\mathbb{R}^3$ are linearly dependent.

$* * *$ **This is the end of the exam.** $* * *$