Math 18 Final Exam
Spring 2007

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Read these instructions carefully.

- Write your name, section number, and "Math 18 Final Exam" on the front of a blue examination book. (Noon is section 1, 2pm is section 2.)
- Read each problem carefully before you attempt to solve it.
- Write your solutions to the problems in the examination book. **Show your work!**
- Clearly indicate where your solution to each problem begins and ends.
- Make sure your solutions are clear, concise, and legible.

Do not turn the page until you are told to do so!

1. Find the line containing the origin in $\mathbb{R}^3$ that perpendicularly intersects the line

$$\mathbf{l}(t) = \langle -3, 4, 6 \rangle + t\langle 2, -1, -3 \rangle.$$

2. Let $f(x, y) = x^2y + e^{x+2y}$.
   (a) Compute the gradient $\nabla f(x, y)$.
   (b) Compute the directional derivative of $f(x, y)$ at $(0, 0)$ in the direction of $\mathbf{v} = \langle 1, 1 \rangle$.
   (c) Find a unit vector $\mathbf{u}$ such that the directional derivative $D_{\mathbf{u}} f(0, 0)$ is zero.

3. Let $g(x, y) = x^2 + 2y^2$.
   (a) Find and classify the critical points of $g(x, y)$.
   (b) Find the absolute maximum and minimum values of $g(x, y)$ on the disk $x^2 + y^2 \leq 1$.

4. Let $F(x, y) = x^2y + y^2$.
   (a) Compute $\displaystyle\iint_R F(x, y)\, dA$ where R is the square $[0, 1] \times [0, 1]$.
   (b) Compute $\displaystyle\iint_D F(x, y)\, dA$ where D is the unit disk $x^2 + y^2 \leq 1$.

5. Let E be the set of points in $\mathbb{R}^3$ that lie above the cone $z = \sqrt{x^2 + y^2}$, outside the sphere $x^2 + y^2 + z^2 = 1$, and inside the sphere $x^2 + y^2 + z^2 = 4$.
   (a) Sketch the region E.
   (b) Compute the volume of E.
   (c) Find the centroid of E.

6. For each of the following vector fields, either find a potential function or prove that the vector field is not conservative.
   (a) $\mathbf{F}(x, y) = e^{y^2}\mathbf{i} + 2xye^{y^2}\mathbf{j}$
   (b) $\mathbf{F}(x, y) = \langle 2xy - y^3, x^2 - 2xy^2 \rangle$
   (c) $\mathbf{F}(x, y, z) = \langle ye^{xy} + yz, xz, xy \rangle$

7. Let $\mathbf{F}(x, y, z) = \langle -\cos(z), \sin(z), 0 \rangle$
   (a) Show that $\nabla \times \mathbf{F} = \mathbf{F}$ (that is, $\mathbf{F}$ is equal to its own curl).
   (b) Let S be the unit sphere centered at $(0, 0, 0)$ in $\mathbb{R}^3$ with the outward orientation. Compute $\displaystyle\iint_S \mathbf{F} \cdot d\mathbf{S}$.

8. Let T be the part of the plane $x + y + z = 1$ that lies in the first octant of $\mathbb{R}^3$. Orient T so that the normal has positive z component. Let $\mathbf{F}(x, y, z) = \langle x^3y^2z, -x^2y^3z, 6z^2 \rangle$.
   (a) Compute $\displaystyle\iint_T \mathbf{F} \cdot d\mathbf{S}$.
   (b) Compute $\displaystyle\oint_{\partial T} \mathbf{F} \cdot d\mathbf{r}$.