

**Math 18 Exam 2**

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**Read these instructions carefully.**

- Write your name, section number, and “Math 18 Exam 2” on the front of a blue examination book. (Noon is section 1, 2pm is section 2.)
- Read each problem carefully before you attempt to solve it.
- Write your solutions to the problems in the examination book. **Show your work!**
- Clearly indicate where your solution to each problem begins and ends.
- Make sure your solutions are clear, concise, and legible.

**Do not turn the page until you are told to do so!**

1. Let  $f(x, y) = x^3 + y^3 - 3x - 3y + 9$ .
  - (a) Compute the gradient  $\nabla f(x, y)$ .
  - (b) Find and classify the critical points of  $f$ .
  
2. Let  $D$  be the type  $I$  region in  $\mathbb{R}^2$  between the graphs of  $y = \cos(x)$  and  $y = -\cos(x)$  for  $0 \leq x \leq \frac{\pi}{2}$ .
  - (a) Sketch the region  $D$ .
  - (b) Write an iterated integral that represents the area of  $D$ , and then compute this area.
  - (c) Find the centroid of  $D$ .
  
3. Let  $Q$  be the region in the first quadrant of  $\mathbb{R}^2$  between the circles  $x^2 + y^2 = 1$  and  $x^2 + y^2 = 4$ .
  - (a) Sketch the region  $Q$ .
  - (b) Let  $g(x, y) = \frac{x e^{x^2+y^2}}{\sqrt{x^2+y^2}}$ . Compute  $\iint_Q g(x, y) dA$ .
  
4. Let  $T$  be the solid tetrahedron in  $\mathbb{R}^3$  bounded by the three coordinate planes and the plane  $x + y + z = 1$ .
  - (a) Compute the volume of  $T$ .
  - (b) Suppose that  $T$  represents an object whose density is given by  $\rho(x, y, z) = z$ . Compute the mass of the object.
  
5. Compute the volume of the solid ellipsoid

$$E = \left\{ (x, y, z) \mid \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1 \right\}$$

where  $a, b, c > 0$ .