Math 18 Exam 1
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Read these instructions carefully.

- Write your name, section number, and "Math 18 Exam 1" on the front of a blue examination book. (Noon is section 1, 2pm is section 2.)

- Read each problem carefully before you attempt to solve it.

- Write your solutions to the problems in the examination book. **Show your work!**

- Clearly indicate where your solution to each problem begins and ends.

- Make sure your solutions are clear, concise, and legible.

Do not turn the page until you are told to do so!

1. Find the line containing the origin in $\mathbb{R}^3$ that perpendicularly intersects the line

$$\mathbf{l}(t) = \langle 3, 5, 1 \rangle + t \langle 2, 4, 3 \rangle.$$

2. Consider the parameterized curve $\mathbf{r}(t) = \langle t^2, \frac{1}{3}t^3, -t^2 \rangle$.

    (a) Find the unit tangent vector at $t = 2$.

    (b) Find the arc length of the curve between $t = 0$ and $t = 1$.

3. Let $f(x, y) = e^{-x^2 - 2y^2}$.

    (a) Find the domain of f.

    (b) Find the range of f.

    (c) Draw a contour plot of the function f showing several level curves.

4. Find the limit or show that it does not exist.

    (a) $\displaystyle\lim_{(x,y) \to (0,0)} \frac{x^2}{x^2 + y^2}$

    (b) $\displaystyle\lim_{(x,y) \to (0,0)} \frac{xy(x - y)}{x^3 + y^3 + (x - y)^3}$

5. Let $F(x, y) = \sqrt{4 - x^2 - 2y^2}$.

    (a) Compute the partial derivatives $\dfrac{\partial F}{\partial x}$ and $\dfrac{\partial F}{\partial y}$.

    (b) Find an equation of the tangent plane to the surface $z = F(x, y)$ at the point $(1, -1, 1)$.

    (c) Is F differentiable at $(1, -1)$?