Math 104 Homework 2

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(1) **Finite Projective Planes.** Recall that a *finite projective plane* is a projective plane with finitely many points; in other words, it is a model of incidence geometry in which:

- The set of points is finite
- Each line is incident with at least three points
- The elliptic parallel property holds (there are no parallel lines)

Throughout this exercise, let $\mathscr{P}$ represent a finite projective plane.

(a) Let l be a line in $\mathscr{P}$ that contains exactly n points; let P be a point not on l. Show that exactly n lines contain P (i.e. *at least* n lines contain P, and *no more* than n lines contain P).

(b) Let P be a point in $\mathscr{P}$ that is contained in exactly k lines; let l be a line that does not contain P. Show that l contains exactly k points.

(c) Show that every line in $\mathscr{P}$ contains the same number of points.

(d) Show that every point in $\mathscr{P}$ is contained in the same number of lines.

(e) Show that the number of points on each line of $\mathscr{P}$ is equal to the number of lines through each point of $\mathscr{P}$. (This should follow easily from the previous propositions.)

If this number is $(N + 1)$, we say $\mathscr{P}$ is a finite projective plane of *order* N. For example, the Fano plane has order 2.

(f) Show that the total number of points in $\mathscr{P}$ is $N(N + 1) + 1$, where N is the order of $\mathscr{P}$. (Count using the set of lines through some point P.)

(g) Show that the total number of lines in $\mathscr{P}$ is $N(N + 1) + 1$, where N is the order of $\mathscr{P}$. (Count using the set of points on some line l.)

[Note: Parts (a) and (b) form the foundation of this problem; everything else should follow from them with relative ease.]

(2) **Betweenness**. Problems from Chapter 2 of Greenberg:

- Exercise 1 on p. 104
- Exercise 3 on p. 104
- Exercise 4 on p. 104