Math 104 Homework 11 (The Last)

David Dumas

Due Thursday, May 4, 2006

(1) **Getting there.** Let A and B be points in a model of neutral geometry.
  (a) Show that there is a rotation R such that $R(A) = B$.
  (b) Show that there is a translation T such that $T(A) = B$.
  (c) Show that if the plane is hyperbolic, then there is a parabolic isometry P such that $P(A) = B$.

  Thus there are several ways to "get from A to B".

(2) **Squares.** Let Q denote the Euclidean square in $\mathbb{R}^2$ with vertices $(0, 1)$, $(0, 2)$, $(1, 2)$, and $(1, 1)$. Now consider Q as a subset of the upper half-plane model of the hyperbolic plane.
  (a) Describe the four sides of Q in terms of hyperbolic geometry. (i.e. What kinds of curves are they? Are they line segments? Hypercycles? Something else?)
  (b) Let $F : H \to \Delta_P$ denote the isomorphism from the upper half-plane model of $\mathbb{H}^2$ to the Poincaré disk model. Draw a picture of $F(Q)$.

  For each pair of integers (i, j), let $Q_{i,j}$ denote the Euclidean square in $\mathbb{R}^2$ whose sides are parallel to the x and y axes and have length 2^i, and whose lower left vertex is the point $(j2^i, 2^i)$. For example, $Q_{0,0}$ is the square Q considered above.
  (c) The squares $\{Q_{i,j} \mid i, j \in \mathbb{Z}\}$ "tile" the upper half plane. Draw a picture of this tiling (i.e. draw the squares $Q_{i,j}$ for many values i and j).
  (d) Show that for each i and j, $Q_{i,j}$ and $Q_{0,0}$ are congruent regions in the upper half plane model of $\mathbb{H}^2$. That is, find an isometry T of the upper half plane model of $\mathbb{H}^2$ such that $T(Q_{0,0}) = Q_{i,j}$.

(3) **Finite order isometries.** Let T be an isometry of a model of neutral geometry. Suppose that $T \neq I$ but $T^n = I$ for some integer $n > 1$. Prove that T is either a rotation or a reflection. (Use the classification of isometries, and rule out every other possibility.)

(4) **Conjugation.** Exercise 20 on p. 374 of Greenberg.

(5) **Translations of lines.** Exercise 12 on p. 373 of Greenberg.

(6) *Study for the final exam.*