Math 104 - Fundamental Problems of Geometry

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1. General Information

Web Page	`http://www.math.brown.edu/~ddumas/math104/`
Main Textbook	*Euclidean and Non-Euclidean Geometries: Development and History*, Third Edition, by Marvin Jay Greenberg ISBN 0716724464
Meeting Time	Tues and Thurs 10:30-11:50am (I hour)
Location	Barus & Holley 157
Instructor	David Dumas (`ddumas@math.brown.edu`)
Office	Kassar House 114
Office Hours	Tues 12:00-1:00pm, Wed 2:00-3:00pm, and by appointment
Phone	401-863-7968

2. Course Overview

In this course we will study hyperbolic geometry and its development from classical plane geometry.

We will begin by surveying axiomatic approaches to geometry, from Euclid's postulates to Hilbert's axioms. We will explore geometry without the parallel postulate (neutral geometry) and then geometry with the negation of the parallel postulate (hyperbolic geometry).

We will then shift our focus to a systematic study of the hyperbolic plane through its various models, including the Klein model, the Minkowski space model, and the Poincaré disk model. We will see how lines, circles, angles, and perpendiculars are expressed in these models.

The rest of the course will consist of a selection of topics in hyperbolic geometry, focusing on instructive examples and explicit constructions. The exact set of topics will be determined based on the available time and the background of the enrolled students; some possibilities are listed in the outline below.

3. Topics

(1) Foundations of Geometry
- Euclid's postulates
- The role of the parallel postulate
- Incidence geometry and models

- Hilbert's axioms
- Neutral geometry
- Measure of angle and distance
- The hyperbolic parallel postulate, consequences

(2) Models of the Hyperbolic Plane ($\mathbb{H}^2$)
- Beltrami-Klein projective model
- Poincaré disk model
- Upper half-plane model
- Minkowski model
- Isomorphisms between the models
- Geodesics, parallels, perpendiculars, incidence, angle, and distance in the models

(3) Topics in Hyperbolic Geometry
Selected from:
- Qualitative features of $\mathbb{H}^2$: area, circumference, visual angle
- The isometry group of $\mathbb{H}^2$
- Triangles and hyperbolic trigonometry
- Hyperbolic polygons: existence, angle defect, area
- Ideal and ultra-ideal polygons
- Horocycles, hypercycles, and circles in $\mathbb{H}^2$
- Tilings of $\mathbb{H}^2$
- Riemannian description of $\mathbb{H}^2$
- Pencils of geodesics
- Gluing polygons; hyperbolic surfaces
- The moduli space of pentagons
- Introduction to hyperbolic 3-space

4. Grading

Your final grade for the course will be based on your homework assignments, two quizzes, and a final exam. These components will be weighted as follows:

Homework		20%
Quiz 1	Thu, Feb 23	15%
Quiz 2	Thu, Mar 23	15%
Final Exam	Take-home	50%

The two quizzes will be given in class, while the cumulative final will be a take-home exam handed out at the start of reading period.

5. Homework Policies

Homework will be assigned approximately weekly, with due dates announced at the time of assignment. The problems will involve material from the lectures and from the assigned reading. Do not wait until the last minute to start the homework!

Your lowest homework score will be dropped, and the average of the remaining scores will account for 20% of your course grade.

Late homework will not be accepted. There will be no exceptions to this policy.

6. Attendance

Attending the lectures is mandatory; if you absolutely must miss a lecture, contact the instructor in advance. You must then make separate arrangements to get notes and any class materials from someone else in the class. You are responsible for the contents of all lectures, including any that you cannot attend.

7. Academic Honesty

All Brown University students must adhere to the standards of academic honesty set forth in the Brown Academic Code, which is available from the following URL:

`http://www.brown.edu/Administration/Dean_of_the_College/academic_code/code.html`

Particularly important are the following provisions:

- The homework you submit must be your own work. You may discuss the lectures, course material, and problems with other students, but the solutions you turn in must not be copied from anyone else.
- You must not receive assistance of any kind from other students during the quizzes and the final exam. (For the take-home final, you may consult your notes, course handouts, and the textbook, but may not use other resources or consult other students.)

Suspected violations of these policies will be referred to the Standing Committee. If you are found guilty of academic dishonesty, the consequences are quite severe. Loss of credit in the course and a note on your transcript is a relatively mild but not uncommon response. Temporary or permanent separation from the university may also result.