010402

Math 104 Quiz 2

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Read these instructions carefully.

- Write your name and "Math 104 Quiz 2" on the front of a blue examination book.
- Read each problem carefully before you attempt to solve it.
- Write your solutions to the problems in the examination book.
- Clearly indicate where your solution to each problem begins and ends.
- Make sure your solutions are clear, concise, and legible.

Do not turn the page until you are told to do so!

1. **Circles.** Let δ denote the Euclidean circle with center $(s, 0)$ and radius s, where $0 < s < \frac{1}{2}$.
   (a) Compute the center C and radius S of the hyperbolic circle represented by δ in Δ_P.
   (b) Let γ denote the hyperbolic circle in Δ_P with center $(0, 0)$ and the same hyperbolic radius as δ. Then as Euclidean circles, which has larger radius, δ or γ? Why?

2. **Distorted angles.** Recall that in the Klein model of the hyperbolic plane, lines are represented by Euclidean segments, but the hyperbolic angle between two rays is not necessarily equal to the Euclidean angle.

   Let's call an angle in Δ_K *undistorted* if it is equal to the associated Euclidean angle, and *distorted* otherwise.
   (a) Show that every angle based at $(0, 0) \in \Delta_K$ is undistorted.
   (b) Show that for every point $P \in \Delta_K$, there is an undistorted angle based at P.
   (c) Give an example of a distorted angle in Δ_K (and show that it is distorted).

3. **Ultra-ideal points.** Recall that the pole of a line in Δ_K is an ultra-ideal point, and so a set of lines determines a set of ultra-ideal points. Each part of this problem asks you to draw a picture of the set of ultra-ideal points with a certain property (i.e shade a certain subset of the exterior of the unit circle). *For this problem only, a picture alone is an acceptable solution; a written explanation is not necessary.*
   (a) Draw a picture of the set of all ultra-ideal points that are poles of lines having $(0, 1)$ as an ideal endpoint.
   (b) Let l be the line in Δ_K with ideal endpoints $(-1, 0)$ and $(1, 0)$. Draw a picture of the set of all ultra-ideal points that are poles of lines intersecting l.
   (c) Let $O = (0, 0) \in \Delta_K$ and let γ be a hyperbolic circle in Δ_K with center O. (The radius of γ is not important, but for concreteness you can take it to be $\log \sqrt{3}$ if you want.) Draw a picture of the set of all ultra-ideal points that are poles of lines that intersect γ.
   (d) Let T be the ideal triangle in Δ_K represented by a Euclidean equilateral triangle inscribed in S^1 with $(1, 0)$ as one of its vertices. Draw a picture of the set of all ultra-ideal points that are poles of lines that *do not* intersect T.

4. **Defect.**
   (a) Define the *defect* of a triangle.
   (b) Define the *angle of parallelism function* $\Pi(d)$, and describe its limiting behavior, i.e. $\Pi(0)$ and $\lim_{d\to\infty} \Pi(d)$. (You do not need to write down a formula for $\Pi(d)$.)
   (c) Let T_s denote the triangle in Δ_P whose vertices have polar coordinates $(s, 0)$, $(s, \frac{2\pi}{3})$, and $(s, \frac{4\pi}{3})$, where $0 < s < 1$ (see the figure below). Thus T_s is an equilateral hyperbolic triangle with center $(0, 0)$. Show that the defect of T_s approaches π as $s \to 1$.