010403

Math 104 Final Exam

David Dumas

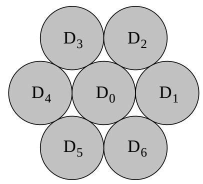
Assigned:	Noon EDT, Friday, May 5, 2006
Due:	4:00pm EDT, Friday, May 12, 2006

Read these instructions carefully before you start to work on the exam.

- Work on the exam **alone**. During the exam period, do not discuss the exam or any course material with other students.
- Refer to your **course notes**, **quizzes**, **homework assignments**, the **textbook**, and the **optional assigned reading** material while working on the exam. All other reference materials (including internet searches and other online resources) are prohibited.
- Write **concise** and **legible** solutions. Clearly indicate where your solution to each problem begins and ends.
- **Write your name at the top of each sheet of paper** you will be turning in, and staple the pages together. Write the total number of pages on the top of the first page.
- Put your completed exam in the **course mailbox in the Kassar House mailroom by 4:00pm EDT on Friday, May 12, 2006**. Late exams will not be accepted.
- Unless you make arrangements with me by e-mail, do not turn in the exam *before* Friday, May 12, 2006.

1. **Convexity of disks.** Recall that an open disk is the set of points inside a circle. Prove in *neutral geometry* that an open disk is a convex set. (Your proof should be pure neutral geometry, without any reference to a particular model.)

2. **Disks around a disk.** In the Euclidean plane, there is a nice arrangement of seven congruent disks $D_0, D_1 \ldots, D_6$, in which $D_1, \ldots, D_6$ form a chain of tangent non-overlapping disks, each of them tangent to D_0, as in the figure below.



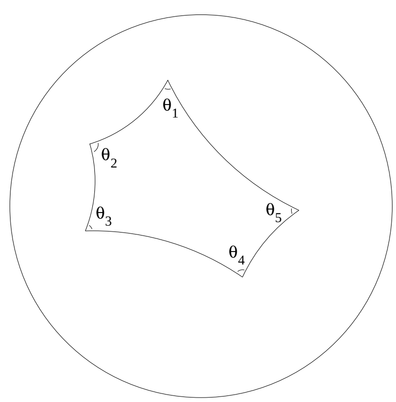
   This problem is about a similar phenomenon in hyperbolic geometry.

   (a) Show that this pattern of tangencies *cannot* be realized by seven congruent disks in the hyperbolic plane.

   (b) For any integer $n > 6$, show that there is a real number $R_n > 0$ such that a hyperbolic disk D_0 of radius R_n can be surrounded by a chain of tangent non-overlapping disks $D_1, \ldots, D_n$ of radius R_n, each of them tangent to D_0.
   More precisely, this means that $D_0, D_1, \ldots, D_n$ are non-overlapping and satisfy the following conditions:
   - D_i is tangent to D_0 for $i = 1, \ldots, n$
   - D_i is tangent to D_{i+1} for $i = 1, \ldots, (n-1)$
   - D_n is tangent to D_1

   (You might start by drawing the case $n = 7$ or $n = 8$ in the Poincaré disk model to convince yourself that it is possible.)

3. **Area of polygons in $\mathbb{H}^2$.** Let P be a *convex* n-gon in $\mathbb{H}^2$ with internal angles $\theta_1, \ldots, \theta_n$. (See the figure below for an example with $n = 5$.) Compute the area of P in terms of the angles θ_i.



4. **Conjugation.** Let S and T be isometries of a model of neutral geometry.

   (a) Show that T and STS^{-1} have the same type (i.e. reflection, rotation, translation, parabolic, glide).

   (b) Show that if T is a translation with translation distance d, then STS^{-1} has translation distance d.

   (c) Show that if T is a rotation with rotation angle θ, then STS^{-1} has rotation angle θ.