0052012

Math 52 Exam 2

Fall 2005

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Name (print legibly!): ____

Answer the questions in the spaces provided on the question sheets. If you run out of room, continue on the back of the page.
Show your work and circle your final answer.

The exam has **4** questions and **9** pages, including this cover page.

Question	Points	Score
1	28	
2	25	
3	20	
4	24	
Total:	97	

Do not open the exam until you are told to do so!

1. Consider the 3×4 matrix

$$A = \begin{pmatrix} 1 & 1 & 2 & 3 \\ 1 & 2 & 3 & 1 \\ 0 & -1 & -1 & 2 \end{pmatrix}.$$

In this problem you will need to find bases for the four subspaces of A and solve a linear system $A\mathbf{x} = \mathbf{b}$.

(a) (4 points) Which subspaces are the same for A and its reduced row echelon matrix R? **Circle the statement(s) that are true.**

i. $C(A) = C(R)$

ii. $N(A) = N(R)$

iii. $C(A^T) = C(R^T)$

iv. $N(A^T) = N(R^T)$

(b) (5 points) Compute the rank of A and the dimensions of the four subspaces. Put your answers in this table:

$\text{rank}(A)$	$\dim C(A)$	$\dim N(A)$	$\dim C(A^T)$	$\dim N(A^T)$

This page is a continuation of problem 1.

$$A = \begin{pmatrix} 1 & 1 & 2 & 3 \\ 1 & 2 & 3 & 1 \\ 0 & -1 & -1 & 2 \end{pmatrix}.$$

(c) (4 points) Find a basis for $C(A)$.

(d) (4 points) Find a basis for $N(A)$.

(e) (4 points) Find a basis for $C(A^T)$.

(f) (4 points) Find a basis for $N(A^T)$.

This page is a continuation of problem 1.

$$A = \begin{pmatrix} 1 & 1 & 2 & 3 \\ 1 & 2 & 3 & 1 \\ 0 & -1 & -1 & 2 \end{pmatrix}.$$

(g) (3 points) Find the **general solution** to $A\mathbf{x} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$. If there are no solutions at all, write "no solutions".

2. Let V be the subspace of $\mathbb{R}^4$ spanned by the vectors $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$, and $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$.

(a) (5 points) What is the dimension of V?

(b) (5 points) Find a basis for V.

(c) (5 points) Find a basis for the orthogonal complement $V^{\perp}$.

This page continues problem 2; V is the span of $\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$, $\begin{pmatrix}0\\1\\1\\0\end{pmatrix}$, and $\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$.

(d) (5 points) Find the projection of $\mathbf{b} = \begin{pmatrix}0\\2\\0\\0\end{pmatrix}$ onto V.

(e) (5 points) Find the projection of $\mathbf{b} = \begin{pmatrix}0\\2\\0\\0\end{pmatrix}$ onto $V^\perp$.

3. Let $A = \begin{pmatrix} -2 & 4 \\ 1 & 1 \\ 2 & -1 \end{pmatrix}$.

(a) (10 points) Use the Gram-Schmidt algorithm to find an orthonormal basis $\mathbf{q}_1, \mathbf{q}_2$ for the column space of A.

(b) (10 points) There is no solution to $A\mathbf{x} = \mathbf{b}$ when $\mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$. Find the best approximate solution $\hat{\mathbf{x}}$, which minimizes $\|A\hat{\mathbf{x}} - \mathbf{b}\|$.

4. Using cofactors, the big formula, pivots, or any other (valid) method, compute the following:

(a) (4 points) $\begin{vmatrix} 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{vmatrix}$

(b) (4 points) $\begin{vmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix}$

(c) (4 points) $\begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & 3 \\ 1 & -2 & 1 \end{vmatrix}$

This page is a continuation of problem 4.

(d) (4 points) $A = \begin{pmatrix} 0 & 3 & 0 \\ 2 & 0 & 2 \\ -2 & 0 & 2 \end{pmatrix}$. What is $|A^{-1}|$?

(e) (4 points) A is invertible. What is $|A^T(A^{-1})^2A^T|$?

(f) (4 points) Q is orthogonal. What is $|Q^{2006}|$?