0052021

Math 52 Exam 1

David Dumas

Name (print legibly!): ______________________________

Answer the questions in the spaces provided on the question sheets. If you run out of room, continue on the back of the page. **Show your work.**

The exam has **4** questions and **9** pages, including this cover page.

Question	Points	Score
1	20	
2	20	
3	30	
4	20	
Total:	90	

Do not open the exam until you are told to do so!

1. Suppose A is a 3×3 matrix with the property that (row 3) $= 5$(row 1) $+ 11$(row 2).

   (a) (5 points) Explain why there is no solution to $A\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

   (b) (5 points) What condition on b_1, b_2, b_3 is necessary for $A\mathbf{x} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ to have a solution?

   (c) (5 points) Explain why A is not invertible.

(d) (5 points) Given that (row 3) = 5(row 1) + 11(row 2), does

$$A\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 16 \end{pmatrix}$$

necessarily have a solution? If so, explain why. If not, give an example of a matrix A meeting the condition but for which there is no solution.

2. The three parts of this question concern the 3×3 matrix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & -1 \\ -2 & 3 & -9 \end{pmatrix}$$

(The matrix is repeated at the top of each page that is part of this question.)

(a) (10 points) Find the matrices L (lower triangular) and U (upper triangular) in the decomposition $A = LU$.

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & -1 \\ -2 & 3 & -9 \end{pmatrix}$$

(b) (5 points) What is the rank of A?

(c) (5 points) Describe the null space $N(A)$.

3. The four parts of this question concern the matrix

$$A = \begin{pmatrix} -1 & 3 & 11 & 7 \\ 1 & 6 & 7 & 20 \end{pmatrix}$$

(The matrix is repeated at the top of each page that is part of this question.)

(a) (5 points) Describe the column space $C(A)$. (Don't just state the definition, describe it for this particular matrix.)

(b) (10 points) Find the reduced row echelon form R for A.

$$A = \begin{pmatrix} -1 & 3 & 11 & 7 \\ 1 & 6 & 7 & 20 \end{pmatrix}$$

(c) (5 points) What is the rank of A?

(d) (10 points) Describe the null space $N(A)$. (Describe it as the set of linear combinations of a few vectors.)

4. Let $\mathbf{M}_{2\times 2}$ be the vector space of all 2×2 matrices. An element of $\mathbf{M}_{2\times 2}$ looks like:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

Determine whether or not each of the following subsets of $\mathbf{M}_{2\times 2}$ is a subspace.

(a) (5 points) The matrices with $a_{11} \neq 0$.

(b) (5 points) The matrices with $a_{11} = 0$.

(c) (5 points) The matrices with at least one entry equal to zero.

(d) (5 points) The matrices with $a_{11} \geq a_{12}$.