

## Summer Tutorial: Groups of Möbius Transformations

**Abstract:** We will discuss the geometric, analytic, and topological aspects of Kleinian groups at an introductory level. The main focus of the course will be detailed analysis of interesting families of examples, motivated by the basic question:

**What can be said about the group generated by two Möbius transformations,**  
 $f(z) = \frac{az+b}{cz+d}$  **and**  $g(z) = \frac{a'z+b'}{c'z+d'}$  **?**

Our journey will reveal a few of the stunning intricacies and subtleties hidden in this seemingly simple question, while allowing us to marvel at the geometric wonders encountered along the way.

**Text:** *Indra's Pearls: An Atlas of Kleinian Groups*, the forthcoming book by David Mumford, Caroline Series, and David Wright, has many beautiful color pictures of limit sets and parameter spaces of Kleinian groups. Though it is intended for a popular audience (and assumes little knowledge of mathematics), we will use this book as the main text, supplementing it with lecture notes and material from other sources to fill out the mathematical background of the concepts covered therein.

### Outline of topics to be covered:

- (1) Möbius transformations and geometry of  $\hat{\mathbb{C}}$
- (2) Symmetries of tilings of  $S^2$  and  $\mathbb{R}^2$
- (3) Tilings of  $\mathbb{H}^2$ , introduction to Fuchsian groups
- (4) Limit sets and domains of discontinuity
- (5) Comparison with complex dynamics
- (6) Schottky groups
- (7) Maskit's construction of punctured tori
- (8) Exploration near the boundary of the Maskit slice
- (9) Methods for drawing pictures
- (10) Connection with 3-dimensional hyperbolic geometry

### Sample student project topics:

- The classification of elementary Kleinian groups
- Maskit-type constructions for multiply punctured tori, higher genus
- Arithmetic examples of Kleinian groups
- Jørgensen's inequality and a naïve discreteness algorithm

**Prerequisites:** Knowledge of complex analysis at the level of Math 113, and the basics of differentiable manifolds.

As summer tutorials are typically small, there is a great opportunity to tailor the level, pace, and content to the common background of a group of students, working from the basic outline and ideas above. Interested students concerned about prerequisites should contact me.

### References:

- *Indra's Pearls : An Atlas of Kleinian Groups*, David Mumford, Caroline Series, and David J. Wright, Cambridge University Press, 2002. ISBN: 0521352533
- *Geometry of Discrete Groups*, Alan F. Beardon, Springer Verlag, 1983. ISBN: 0387907882
- *Kleinian Groups*, Bernard Maskit, Springer Verlag, 1998. ISBN: 0387177469

David Dumas  
ddumas@math.harvard.edu