Hyperbolic Geometry in Two and Three Dimensions

Abstract: We will discuss hyperbolic geometry in dimensions two and three at an introductory level, starting with the basics of geometry in the hyperbolic plane and hyperbolic 3-space. The intrinsic geometry of closed hyperbolic surfaces will be emphasized, with detailed investigation of specific examples. We will also give a flavor of the vast field of hyperbolic 3-manifolds through gluings of polyhedra and link complements.

Outline of topics to be covered:

(1) The hyperbolic plane, its metric, various models thereof
(2) Hyperbolic geodesics, angles, areas, polygons
(3) The group of hyperbolic isometries, classification
(4) Construction of hyperbolic surfaces
(5) Comments on Hyperbolic vs. Riemann surfaces vs. algebraic curves
(6) Geodesics and isometries of hyperbolic surfaces
(7) Examples of hyperbolic surfaces
(8) Hyperbolic 3-space, isometries, geodesics, hyperplanes
(9) The sphere at infinity, connection with conformal geometry
(10) Examples of hyperbolic 3-manifolds

Some possible student project topics:

- Construction of hyperbolic polyhedra (Andreev's theorem)
- Tilings of the hyperbolic plane and their symmetry groups
- Geodesics on hyperbolic punctured tori
- Hyperbolic surfaces with many automorphisms
- Volumes of knot complements
- Convex geometry in hyperbolic space
- Hyperbolic manifolds with boundary

Prerequisites: Some familiarity with topology and differential geometry of smooth manifolds (especially closed surfaces). Previous exposure to Riemannian geometry would be useful, but is not necessary.

As summer tutorials are typically small, there is a great opportunity to tailor the level, pace, and content to the common background of a group of students, working from the basic outline and ideas above. Interested students concerned about prerequisites should contact me.

References:

- Three dimensional geometry and topology, W. Thurston, Princeton Mathematical Series, 35. Princeton University Press, 1997
- Lectures on hyperbolic geometry, R. Benedetti and C. Petronio, Universitext, Springer-Verlag, 1991
- Foundations of hyperbolic manifolds, J. Ratcliffe, Graduate texts in mathematics 149, Springer-Verlag, 1994

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