Math 535 - Complex Analysis
David Dumas
Spring 2016

1. Basic course information

Course web page	http://dumas.io/math535/
Primary textbook	Lars Ahlfors, *Complex Analysis*, 3ed.
Meeting time	MWF 10:00-10:50am
Location	308 Stevenson Hall
CRN	19436
Instructor	David Dumas (david@dumas.io)
Office	SEO 503
Office Hours	Monday 2-3pm, Wednesday 11am-12pm, or by appointment

2. Course overview

Math 535 is a rigorous introduction to the theory of functions of one complex variable at the graduate level. This fundamentally important subject has applications in nearly every branch of mathematics (algebra, number theory, combinatorics, algebraic geometry, differential geometry, and numerical analysis, to name a few).

It will be assumed that students are familiar with the field of complex numbers. Some students entering Math 535 will have taken an undergraduate complex analysis course. While prior exposure to the subject at an undergraduate level might be useful, it is not required.

Our first topic will be the concept of differentiability for complex-valued functions of a complex variable (i.e. *complex functions*). Unlike the case of real-valued functions, it turns out that the existence of one derivative in the complex sense implies infinite differentiability and analyticity (representation by a convergent power series). This follows from the most important theorem in complex analysis—the *Cauchy integral formula*. Roughly speaking, this theorem implies that complex differentiation and complex integration are equivalent.

A substantial part of the course will be devoted to understanding the relationship between a complex function, its singularities, and its integrals along curves in the complex plane. This comprises the theory of complex integration and the *calculus of residues*. Remarkably, we will see that some integrals of real-valued functions are most easily evaluated using these complex-analytic considerations.

The set $\mathbb{C}$ of complex numbers can be identified with the Euclidean plane $\mathbb{R}^2$, and complex functions can be seen as geometric objects that transform this plane. Taking this point of view, we will study differentiable mappings between open sets in the complex plane (individually and in families). This discussion will include a proof of the *Riemann mapping theorem*, which asserts that any proper open subset of $\mathbb{C}$ that is topologically equivalent to the unit disk is actually complex-analytically equivalent to the disk. We will discuss applications of this theorem and work through some examples of such *conformal mappings*.

Complementing our discussion of power series, which are infinite sums, we will also study the representation of holomorphic functions as *infinite products*. Such representations generalize the factorization of a polynomial function $p(z)$ into finitely many linear terms $(z-a_i)$ corresponding to its zeros. We will construct and study infinite product expressions for several important holomorphic functions.

Finally, we will discuss some special examples and classes of holomorphic functions which are important for applications to other areas of mathematics. These include the Riemann zeta function, elliptic functions (including the Weierstrass function $\wp$), and the the modular invariants λ and j.

The planned course material corresponds to chapters 1–5 and selected topics from chapters 6–8 in the textbook.

3. Grading

The final grade for the course will be based on the homework assignments, an in-class midterm exam, and a cumulative final exam. These components will be weighted as follows:

- 40% Homework
- 20% Midterm (Monday, February 29, in class)
- 40% Final exam (Friday, May 6, 10:30am-12:30pm)

4. Homework Policies

There are two types of homework: **Weekly problems** and **challenge problems**.

Weekly homework will be assigned on the course web page, and will consist mostly of problems from the textbook.

Weekly homework is due each Monday by 4pm and should be submitted directly to the mailbox of the grader, John Kopper, in the 3rd floor SEO mailroom.

A list of challenge problems is posted on the course web page and updated regularly. During the semester, students are required to complete at least **four** of these problems and submit written solutions. At least two solutions must be submitted before the midterm exam. Solutions to these problems will be held to a high standard of completeness, clarity, and correctness; do not be alarmed if a solution to one of the challenge problems is returned with comments and re-submission is requested.

Challenge problems can be submitted directly to the instructor at any time.

Students are allowed (and encouraged) to study the course material and work on the homework with other students. However, each student must:

(1) Write and submit their own solutions
(2) Acknowledge collaborators by name on the assignment (e.g. write "in collaboration with Jane Doe" at the top of the page).

The homework grade will be determined by dropping the lowest weekly assignment score and then averaging the remaining weekly scores and the four required challenge problems. In particular, *each challenge problem is worth as much as a weekly homework assignment.*

Students are encouraged to complete and turn in more than the required number of challenge problems. Doing so will have a modest positive effect on the homework grade in a manner to be determined before the end of the semester.

5. Exams

There will be an in-class midterm exam on Monday, February 29, during the regular lecture time (10:00-10:50am in 308 Stevenson Hall).

The final exam will be held at the time set by the registrar: Friday, May 6, 10:30am-12:30pm. The final exam location will be announced at a later date.

6. Participation

Students are encouraged to ask questions in lecture about the material currently under discussion, and to answer questions asked by the instructor.

Questions specific to a single student (such as inquiries about how an exam problem was graded, etc.), are better left to office hours.

7. Attendance

Students are responsible for all of the material covered in the lectures, including any lectures they miss. Any student who misses a lecture is advised to ask classmates for notes and information about any assignments or course announcements. Lecture notes are not provided by the instructor (in case of absence or otherwise).

Frequent absence from lecture is inconsiderate and moreover such absences generally lead to poor performance on homework and exams.

8. Academic Honesty

All UIC students (graduate and undergraduate) are required to maintain the standards of academic integrity described in the *Guidelines Regarding Academic Integrity*:

http://dos.uic.edu/docs/Guidelines%20for%20Academic%20Integrity.pdf

In particular, this policy prohibits plagiarism. Any violation of these standards will be handled in accordance with the Student Disciplinary Policy.

9. Disability Accommodation

The University of Illinois at Chicago is committed to maintaining a barrier-free environment so that students with disabilities can fully access university programs, courses, services, and activities. Students with disabilities who require accommodations for access or participation in this course are welcome, but must be registered with the Disability Resource Center (DRC). Students may contact the DRC at 312-413-2183 (voice) or 312-413-0123 (TTY). Further information is available from the DRC web page: http://drc.uic.edu/