Math 535: Complex Analysis – Spring 2016 – David Dumas

Midterm Exam

Instructions:

- Complete **three** of the problems below.
- Each problem is worth 10 points.
- If you complete more than three problems (which is *not* recommended) your score will be the sum of your three best problem scores.
- None of these problems require long, complicated calculations.

Problems:

(1) Is it possible to conformally map the punctured complex plane $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$ to the annulus $A = \{z : 1 < |z| < 2\}$? If it is possible, find such a conformal map. If not, prove that it is impossible.

(2) Suppose $a,b,c,d \in \mathbb{C}$. Show that the following conditions are equivalent:

(i) There exists a linear fractional transformation S such that Sa, Sb, Sc, Sd are the vertices of a square, listed in counter-clockwise order.

(ii) $\dfrac{(a-c)(b-d)}{(b-c)(a-d)} = 2.$

(3) Suppose that f is analytic on a region Ω and that $|f(z)-1| < 1$ for all $z \in \Omega$. Show that

$$\oint_\gamma \frac{f'(z)}{f(z)}\, dz = 0$$

for every closed curve $\gamma \subset \Omega$.

(4) Let γ be the circle $|z| = 2016$ oriented counter-clockwise. Calculate:

$$\oint_\gamma \frac{e^z - 1}{z}\, dz$$

(5) Suppose f is holomorphic and has a simple zero at $z=0$. (That is, $f(0)=0$ and $f'(0) \neq 0$.) Show that there does *not* exist a function g holomorphic in an open disk containing 0 such that $f(z) = g(z)^2$ on their common domain.