Math 535: Complex Analysis – David Dumas – Spring 2016

Homework 8

Due Monday, March 7 at 4:00pm

(—) From the textbook: Section 4.4.7 (p148): 2, 4, 5

Note: We will probably discuss part of problem 4 in lecture; you must still write a complete, correct solution that fills in any details omitted in that discussion.

(P1) A region $\Omega \subset \mathbb{C}$ is called *star-shaped* if there exists $z_0 \in \Omega$ such that for all $z \in \Omega$ the closed line segment with endpoints z_0, z is contained in Ω. (The idea is that Ω looks like a "star" with rays emanating from z_0.)

Show that any star-shaped region is simply connected.

(P2) Suppose f is analytic in $\Delta^* = \{z : 0 < |z| < 1\}$.

(a) Give an example to show that, in general, such f need not be the derivative of an analytic function defined in a punctured neighborhood of 0.

(b) Show that one can "correct" f by adding a multiple of $1/z$ so that it becomes the derivative of an analytic function. That is, prove:

Lemma: There exists $a \in \mathbb{C}$ and analytic function F on Δ^* such that

$$F'(z) = f(z) - \frac{a}{z}.$$