

Two-Generator Kleinian Group Example

David Dumas

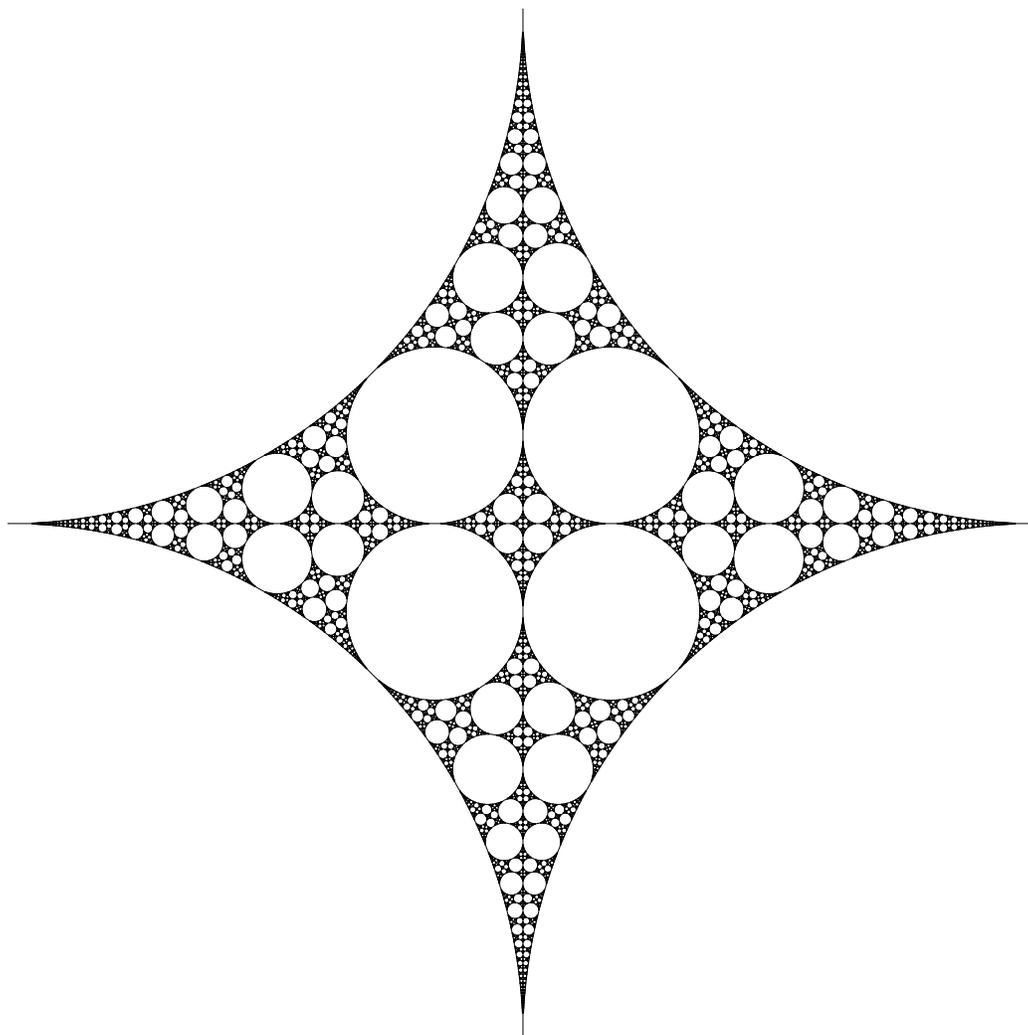
Consider the two Möbius transformations

$$P(z) = (3 + 2\sqrt{2}) \frac{z - 1}{z + 1},$$

$$R(z) = iz.$$

Let Γ denote the group generated by these transformations. The elements of Γ are all transformations obtained by composing P , R , and their inverses, in any sequence.

The figure below shows the Γ -orbit of the unit circle S^1 in the complex plane. The orbit consists of all of the circles obtained by applying the transformations in Γ to S^1 .



Note that the visible portion of the complex plane in the figure above is the square of side length $2\sqrt{2}$ centered at the origin. The complete orbit extends beyond this square, and contains an infinite sequence of nested figures that look like the one above. Similarly, one can see that the figure contains a smaller copy of itself at the center (which in turn contains a smaller one, etc...).

The figure was created with Curt McMullen's program *lim*, which is available at:

<http://www.math.harvard.edu/~ctm/programs.html>